MULTI-SAMPLE ANALOGUES OF SOME ONE-SAMPLE TESTS

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Preface. The results of Part I and Part III were obtained by the second author (cf. Puri (1962)) and those of Part I and Part II by the first author by following different methods (cf. Mehra (1963)). The authors wish to express their sincere thanks to Professors Erich L. Lehmann, Jaroslav Hájek and Edward Paulson for very helpful suggestions and criticisms.

PART I

I.1. Introduction and summary. Consider K treatments in an experiment which yields paired observations, namely (X_{il}, X_{jl}), $l = 1, \cdots, N_{ij}$; $1 \leqq i < j \leqq K$, obtained by N_{ij} independent paired comparisons for each pair (i, j) of treatments and assume that N_{ij} difference scores $Z_l^{(i,j)} = X_{il} - X_{jl}$, $l = 1, \cdots, N_{ij}$, have a common continuous cdf (cumulative distribution function) $\Pi_{ij}(z)$. This is the situation, for example, if in the analysis of an incomplete blocks experiment with each block of size two, one makes the assumption of additivity in the usual analysis of variance model. Then for testing the hypothesis

$$H_0 : \Pi_{ij}(z) + \Pi_{ij}(-z) = 1 \quad \text{and} \quad \Pi_{ij}(z) = \Pi_{i'j'}(z)$$

for any two pairs (i, j) and (i', j') [which states that each of the distributions Π_{ij} of the differences $Z_{ijl} = X_{il} - X_{jl}$, $l = 1, \cdots, N_{ij}$, is symmetric with respect to the origin, and furthermore all distributions Π_{ij} are identical] some rank tests based on the generalizations of the one-sample Chernoff-Savage-Hájek type tests (cf. [9] and [3]) are proposed, their limiting distributions are derived, and their efficiency properties with respect to one another and some of their competitors, viz. the Bradley-Terry test [1], the Durbin test [6] and the classical F test are studied. (For alternative formulations of the null hypothesis, and the study of the special case of the generalization of the one-sample Wilcoxon test, the reader is referred to [16].)

Let $\{J_{N,k}; k = 1, \cdots, N\}$, be a double sequence of numbers satisfying certain conditions to be stated below (Section 2) and let $R_{N,l}^{(i,j)}$ be the rank of $|Z_l^{(i,j)}|$, when the $N = \sum_{i=1}^{k} \sum_{j>i} N_{ij}$ absolute values of the observed differences $|Z_l^{(i,j)}|$, $l = 1, 2, \cdots, N_{ij}$, $1 \leqq i < j \leqq K$, are arranged in the ascending order of

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