LOCALLY MINIMAX TESTS[1]

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1. Introduction. This paper represents an extension of the local minimax results contained in Section 2 of Giri and Kiefer (1964), hereafter G-K (1964). Other sections of G-K (1964) deal with topics other than local minimaxity, with which this paper is not concerned.

In G-K (1964) the property of local minimaxity is defined and Lemma 1 states conditions under which a given test is locally minimax. These conditions on the statistical problem and on the given test are then verified for the settings in which Hotelling's T^2-test and the test based on the squared multiple correlation coefficient, R^2, are customarily employed, showing that the T^2- and R^2-tests are locally minimax.

The present paper deals with the generalizations of the T^2-and R^2-problems, namely, the MANOVA problem and the problem of testing the independence of sets of variates. Whereas both the T^2- and R^2-tests are best fully invariant tests, in both the general MANOVA and independence problems there is a large class of fully invariant admissible tests (Schwartz (1966a, b, c)). Of course different tests within this class may have different contours of constant power. Since the definition of local minimaxity is relative to a family of contours approaching the null hypothesis, it seemed possible at the outset that different fully invariant tests might be locally minimax for different families of contours.

However, examination of the local behavior of the probability ratio of the maximal invariant (under all linear-affine transformations which leave the problem invariant) reveals that in both the MANOVA problem and in testing the independence of *two* sets of variates there is a unique locally best test in the class of fully invariant tests. (These results are given in Theorem 1 and 3 respectively where the meaning of locally best is made clear.) Hence, in both of these problems if any fully invariant test is to be locally minimax it must be the one which is locally best invariant.

Once the locally minimax test has been guessed the verification that it satisfies the conditions of Lemma 1 of G-K (1964) follows very closely the verifications given in G-K (1964) for the T^2- and R^2-tests. The computations are slightly more complicated in the more general settings considered in this paper.

In addition the actual results can have more complicated statements in the more general settings because of the variety of different families of contours which may be considered. Detailed consideration of different families of contours is given only in the MANOVA problem and not in the independence problem. In

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