ABSTRACTS

(Abstracts of paper presented at the Central Regional meeting, Columbus, Ohio, March 23–25, 1967. Additional abstracts appeared in earlier issues.)

34. Generalized multivariate estimators for the mean of a finite population.

PODURI S. R. S. RAO, Information Research Associates, Inc. (Invited)

Just as the ratio estimators are often indicated when the variable y is positively correlated with an auxiliary variable x, so product estimators are indicated when y is negatively correlated with x. For estimating $\bar{Y}$ the following product estimators are considered: $\bar{Y}_1^* = \sum x_i y_i / n\bar{x}$; $\bar{Y}_2^* = \bar{x}\bar{y}/\bar{X}$; $\bar{Y}_3^* = \bar{x}\bar{y}/\bar{X} - (N - n)s_{xy}/Nn\bar{X}$. $\bar{Y}_1^*$ is not a consistent estimator, $\bar{Y}_2^*$ is a biased estimator, and $\bar{Y}_3^*$ is an unbiased estimator. The mean square errors of these estimators are compared when the relation between y and x is of the forms $y = \alpha + \beta x + e$ and $y = \alpha + \beta/x + e$ and when x has a gamma distribution. Similar to the multivariate ratio estimator of Olkin [*Biometrika* **45** (1958), 154–165], multivariate product estimators are formed analogous to the above three estimators and their biases and variances are found. Finally when $x_1, \cdots, x_p$ are positively correlated and $x_{p+1}, \cdots x_q$ are negatively correlated with y, multivariate estimators for $\bar{Y}$ are found and their biases and variances are evaluated. In particular an unbiased multivariate estimator for $\bar{Y}$ is $\tilde{Y} = \tilde{Y}_1 + \tilde{Y}_2$ where

$$\tilde{Y}_1 = \sum W_j r_j \bar{X}_j + [(N-1)n/N(n-1)](\bar{y} - \sum W_j \bar{r}_j \bar{x}_j),$$

$$\tilde{Y}_2 = \sum [W_j/\bar{X}_j][\bar{x}_j\bar{y} - (N-n)(P_j - n\bar{x}_j\bar{y})/Nn(n-1)]$$

with $r_j = y_i/x_{ji}$, $n\bar{r}_j = \sum (y_i/x_{ji})$, $P_i = \sum x_{ji}y_i$. The optimum weights $\hat{W}_j$ are determined by minimizing $V(\tilde{Y})$. (Received 10 April 1967.)

35. The asymptotic distribution of class of two-sample non-linear rank order statistics in the null case.

SIEGFRIED SCHACH, University of Minnesota. (Invited)

Let $X_1, X_2, \cdots, X_m$ and $Y_1, Y_2, \cdots, Y_n$ be two independent samples from the same continuous distribution. Let $h_N(\cdot)$, $N = 1, 2, \cdots$ be a sequence of functions on $[-1, 1]$, symmetric with respect to 0, and periodic with period 1, and let $R_1, R_2, \cdots, R_m$ be the ranks of the X-observations in the combined sample. We find conditions under which a sequence of statistics of the form $T_N = N^{-1} \sum_{i=1}^m \sum_{j=1}^m h_N(R_i - R_j)N^{-1}$ converges in distribution as $N = m + n$ goes to infinity, and we derive the characteristic function of the limiting distribution. If h_N converges to some h with a finite Fourier expansion, the limiting distribution is obtained by writing T_N as a quadratic form in variables $z_i^{(N)}$, where $z_i^{(N)} = 1$ if the ith observation in the ordered combined sample is an X, 0 otherwise, and by diagonalizing the matrix of the quadratic form. Hájek's theorem is used to obtain the limiting distribution of T_N. It is a finite weighted sum of χ^2-variables. If h has an infinite Fourier expansion with an absolutely converging Fourier series, T_N can be represented as a function on the Hilbert H space of real sequences with finite sums of squares. Using Prokhorov's techniques, it is shown that the induced measures on H converge weakly to some Gaussian measure and that T_N has the distribution of an infinite weighted sum of χ^2-variables, where the Fourier coefficients determine the weights. (Received 10 April 1967.)