CONFIDENCE INTERVALS FOR THE MEAN OF A FINITE POPULATION[1]

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1. Introduction. The admissibility of estimates of the population total with squared error as the loss function were considered by the author, (1965), II, and a certain estimate was shown to be always admissible whatever be the sampling design in the entire class of all estimates. This estimate is equivalent to using the sample mean as estimate of the population mean. Here we consider the allied question of admissibility of the confidence intervals for the population mean, based on the sample mean and the sample standard deviation, which are also commonly used in practice. These confidence intervals are here shown to be also always admissible whatever be the sampling design.

Then, by generalizing, the result is also shown to hold for confidence intervals based on a ratio estimate and a generalized version of the sample standard deviation. It is interesting that as shown in a previous paper (1966), IV, Section 5, with squared error as loss function, this ratio estimate is also admissible as an estimate of the population mean, whatever be the sampling design.

2. Notation and definitions. The population U consists of N units $u_1, u_2, \cdots, u_N$; with the unit u_i is associated the variate value x_i, $i = 1, 2, \cdots, N$; $x = (x_1, x_2, \cdots, x_N)$ denotes a point in the Euclidean N-space R_N; a sample s means any subset of U; S denotes the set of all possible samples s; a probability function p is defined on S such that $p(s) \geqq 0$ for all s, and $\sum_{s \varepsilon S} p(s) = 1$. Following Godambe and Joshi (1965), I, the pair (S, p) is called the sampling design. A sample s is drawn from S according to p. Then we have

Definition 2.1. An estimate $e(s, x)$ is a real function e defined on $S \times R_N$ which depends on x through only those x_i for which $u_i \varepsilon s$.

The above definitions of sampling design and estimate are wide enough to cover all sampling procedures and classes of estimates; for a brief account we refer to Godambe and Joshi (1965), I, Section 5.

We next define admissibility of a set of confidence intervals for the population mean,

$$(1) \qquad \bar{X}_N = N^{-1} \sum_{i=1}^{N} x_i .$$

For a given sampling design d, we denote by $\bar{S}$, the subset of S, consisting of all those samples s for which $p(s) > 0$. Now let $e_1(s, x)$, $e_2(s, x)$ be two estimates (Definition 2.1) such that $e_1(s, x) \leqq e_2(s, x)$ for all $x \varepsilon R_N$ and all $s \varepsilon \bar{S}$; then $[e_1(s, x), e_2(s, x)]$ denotes the set of confidence intervals

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