DISTRIBUTION OF THE LARGEST LATENT ROOT AND THE SMALLEST LATENT ROOT OF THE GENERALIZED B STATISTIC AND F STATISTIC IN MULTIVARIATE ANALYSIS

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1. Introduction and results. The cumulative distribution function (cdf) of the largest latent root and the smallest latent root of the generalized B statistic and the generalized F statistic in multivariate analysis has been studied by K. C. S. Pillai [8], [9], [10], [11], and so on. But, since the cdf is represented by the summation of $p!$ incomplete B functions, it is complicated, and we have many difficulties in calculating it. Recently, these general expressions in the following formula were obtained by T. Sugiyama and K. Fukutomi [15], namely the probability element of the largest latent root λ_1 and the smallest latent root λ_p of the generalized B statistic was given respectively by the following formula

$$C\lambda_1^{n_1 p/2-1}(1-\lambda_1)^{(n_2-p-1)/2} \cdot F((-n_2+p+1)/2,\ (n_1-1)/2;\ (n_1+p+1)/2;\ \lambda_1 I_{p-1})\ d\lambda_1$$

and

$$C'\lambda_p^{(n_1-p-1)/2}(1-\lambda_p)^{n_2 p/2-1} \cdot F((-n_1+p+1)/2,\ (n_2-1)/2;\ (n_2+p+1)/2;\ (1-\lambda_p)I_{p-1})\ d\lambda_p$$

where

$$C = \pi^{p/2}B_{p-1}((n_1-1)/2,\ (p+2)/2)/\Gamma(p/2)B_p(n_1/2,\ n_2/2),$$

and

$$C' = \pi^{p/2}B_{p-1}((n_2-1)/2,\ (p+2)/2)/\Gamma(p/2)B_p(n_1/2,\ n_2/2),$$

and also, the probability element of the largest latent root f_1 and the smallest latent root f_p of the generalized F statistic is given by substituting $f_1/(1+f_1)$ for λ_1 and $f_p/(1+f_p)$ for λ_p in above formula respectively. But, since the cdf is also represented by the series of incomplete B function, it is not easy to calculate it. The purpose of this paper is to find the simple general expression of the probability element of the largest latent root λ_1 and the smallest latent root λ_p of the generalized B statistic, and the simple general expression of the cdf of the largest latent root and the smallest latent root of the generalized B statistic and the generalized F statistic which is given by the following theorems, respectively.

THEOREM 1. *Let U_1 and U_2 are two independent matrices having Wishart distribution $W(p, n_1, \Sigma)$ and $W(p, n_2, \Sigma)$ respectively, where $n_1, n_2 \geqq p$. Then the prob-*

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