REPLICATED (OR INTERPENETRATING) SAMPLES OF UNEQUAL SIZES[1]

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1. Introduction.

1.1. The technique of replicated (or interpenetrating) samples, introduced by Mahalanobis, is now well known. Some aspects of this technique, and its underlying theory, relative to equal-sized samples, were clarified by Lahiri (1954) and Koop (1960). Deming (1956) has given a simplified version of the technique based on the use of two or more systematic samples drawn from the entire universe of ultimate units or elements.

1.2. In practice equal-sized replicated samples are used. However if considerations of field work, and/or other causes, should dictate a departure from this practice, the outcome would still be favorable. The main purpose of this paper is to point out this surprising result; that is, replicated samples of unequal sizes are more efficient than those with equal sizes.

2. Technique underlying the theory.

2.1. We consider a single universe U containing N first-stage units

$$u_1, u_2, \cdots, u_j, \cdots, u_N.$$

The first-stage units are selected with equal probabilities, and without replacement after each draw. Beyond this stage, the procedure, say D_j, $(j = 1, 2, \cdots, N)$ for selecting the units at the second and subsequent stages, with equal or unequal probabilities and with or without replacement, is specified in advance for each u_j. For the sake of generality we do not explicitly define this procedure.

2.2. Let X_j $(j = 1, 2, \cdots, N)$ be the total value of a characteristic of interest in the first-stage unit u_j. We note that X_j is equal to the sum of all the variate values in the ultimate units of u_j. It is desired to estimate

$$T = \textstyle\sum_{j=1}^{N} X_j, \tag{1}$$

on the basis of k independent replicated samples each of size $m_i > 1$ $(i = 1, 2, \cdots, k)$. For this purpose, m_1 first-stage units are selected as specified in the previous paragraph. Denote this collection of first-stage units by s_1. Then for every unit u_j, which is a member of s_1, the procedure D_j is applied for selecting the second- and subsequent-stage units and for ascertaining the variate values of the ultimate-stage units. Next these m_1 first-stage units are returned to U. The same procedure is repeated for the selection of sets of $m_2, m_3, \cdots, m_k$ units, and for the selection of the multi-stage units internal to them.

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