ON HITTING PLACES FOR STABLE PROCESSES

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1. Introduction. Throughout this paper $X(t)$ will denote a drift free stable process on R^d (d-dimensional Euclidean space) having transition density $p(t, x)$ and paths which are normalized to be right continuous with left hand limits at every point. The familiar fact that all so normalized processes are strong Markov processes will be used without further explicit mention. For a compact subset $B \subset R^d$, let

$$T_B = \inf\{t > 0: X(t) \varepsilon B\}(= \infty \quad \text{if} \quad X(t) \not\varepsilon B \quad \text{for all} \quad t > 0).$$

be the first hitting time of B. Our purpose in this paper will be to investigate the asymptotic behavior, for large t, of the quantity

$$F(t, x) = \int_B P_x(t < T_B < \infty, \quad X(T_B) \varepsilon dy)f(y), \tag{1.1}$$

where f is a continuous function on B. Previously this quantity was investigated for planar Brownian motion by Hunt [2], and in the special case of $f \equiv 1$ for general stable processes by the author in [4] and [5]. The results we obtain here will be extensions of those for the case $f \equiv 1$ to the case of an arbitrary f, and the proofs of these results will be dependent on the results for the case $f \equiv 1$. In essence, our technique will be to show that the general case can be reduced to the case $f \equiv 1$.

In order to state our results it will be necessary to recall some concepts and notation from [4] and [5]. Here we shall be brief referring the reader to the above cited papers for fuller details.

The measure $P_x(T_B > t, X(t) \varepsilon dy)$ has an upper semi-continuous density $g_B(t, x, y)$ which satisfies the well-known first passage relation:

$$p(t, y - x) - \int_B \int_0^t P_x(T_B \varepsilon ds, \quad X(s) \varepsilon dz)p(t - s, y - z) = g_B(t, x, y). \tag{1.2}$$

Let

$$H_B(x, dy) = P_x(T_B < \infty, X(T_B) \varepsilon dy)$$

and

$$g_B(x, y) = \int_0^\infty g_B(t, x, y)\, dt.$$

We must now discuss the case of recurrent and transient processes separately.

In the recurrent case we assume that $P_x(T_B < \infty) \equiv 1$. Then we know (see [5]) that $g_B(x, y) < \infty$ for $x \neq y$ and $y \not\varepsilon B$. [Actually more is true, but this is all we shall need]. Moreover, except for linear Brownian motion, the limits

$$\lim_{|y| \to \infty} g_B(x, y) = g_B(x, \infty),$$

$$\lim_{|x| \to \infty} \int_B H_B(x, dy)f(y) = \int_B H_B(\infty, dy)f(y)$$

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