

TESTING HYPOTHESES IN RANDOMIZED FACTORIAL EXPERIMENTS¹

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0. Introduction. In the present paper we study the problem of testing the significance of a subgroup of 2^s pre-assigned parameters in an $n/2^{m-s}$ fractional replicate of a 2^m factorial experiment ($s < m$). In our previous paper on Randomization and Factorial Experiments [3] we outlined ANOVA schemes for such tests of significance, which were shown to be appropriate for the two randomization procedures RP I and RP II studied there. The test statistics proposed are the familiar F -like ratios. The main difficulty in performing those tests is in choosing the critical level for a given level of significance. This is also the main problem in performing ANOVA tests in the non-randomized designs. The problem is due to the effects of the nuisance parameters, which may be excessive and yet not under our control.

To be more specific, as will be shown in the sequel, the conditional distribution of the F -like ratio test statistics, given the fractional replicate chosen, is like that of a double non-central $F[\nu_1, \nu_2; \lambda, \lambda^*]$. Here, ν_1 and ν_2 are the appropriate degrees of freedom, λ and λ^* the parameters of non-centrality, being functions of the fractional replicate chosen, and of the vector of unknown parameters. Even under the null hypotheses, that the pre-assigned parameters are zero, λ and λ^* might be quite large due to the effects of the nuisance parameters. In the classical fractional replication model the assumptions imply that, under the null hypotheses, $\lambda = \lambda^* = 0$. For such a model the proper test criterion for level of significance γ is the $(1 - \gamma)$ th fractile of $F[\nu_1, \nu_2]$, i.e., $F_{1-\gamma}[\nu_1, \nu_2]$. This is not the case, however, when λ and λ^* are positive.

When the values of the nuisance parameters are known, the problem is solved by a simple adjustment of the test statistics. A similar adjustment may also yield locally optimal test procedures when certain information is available on the nuisance parameters (see K. Takeuchi [6]). However, no proper solution to the problem can be attained in the non-randomized case if the values of the nuisance parameters are unknown. The objective of the present article is to verify that under certain conditions on the nuisance parameters, if the fractional replicate is chosen according to the randomization procedures studied in [3], the distributions of the F -like test statistics are approximated by the distributions of central F statistics, and the test criterion $F_{1-\gamma}[\nu_1, \nu_2]$ yields approximately the required level of significance γ . We also prove that under the established conditions on the nuisance parameters, if the fractional replicate is of a sufficiently large size com-

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