REDUCED GROUP DIVISIBLE PAIRED COMPARISON DESIGNS

BY J. A. JOHN

University of Southampton

1. Introduction. Suppose that t treatments $1, 2, \cdots, t$ are to be compared in pairs. It will often be impractical to make all possible $\frac{1}{2}t(t-1)$ pairings. Designs are therefore required that reduce the number of comparisons without serious imbalance, that is, give estimates of treatment contrasts with variances as low and as equal as possible.

Incomplete block designs with two treatments per block, i.e. paired comparison designs, given previously fall into two groups. Firstly, the partially balanced incomplete block (PBIB) designs with two associate classes which include the group divisible, the triangular and the square designs. These designs have been completely enumerated by Clatworthy (1955) for $t \leqq 20$ and $2 \leqq r \leqq 10$. Secondly, the cyclic designs. The structure and enumeration of these designs have been given by David (1963) and David (1965).

The purpose of this paper is to produce a class of designs that, in many cases, give more efficient designs than the PBIB or cyclic designs. An efficiency factor will be obtained for each design so enabling comparison to be made with the designs given by Clatworthy (1955) and David (1963). Simplicity of analysis is much less important in the days of electronic computers and, although some of the designs proposed here possess a high degree of symmetry that makes analysis simple, designs have not been constructed with ease of analysis in mind.

The measure of efficiency of paired comparison designs will be obtained from the covariance matrix of the estimates of treatment parameters. The efficiency, E, will be defined as the ratio of the average between treatment variance for the full design to the average between treatment variance for the incomplete design. This is the same measure as used by David (1963), but Clatworthy's figures need to be multiplied by $2(t-1)/t$ to convert them to the values of E.

The designs considered will be of three types. The first two types, A and B, will have the t treatments divisible into m groups of n members each. Let θ_{ip} be the ith treatment in the pth group ($i = 1, 2, \cdots n; p = 1, 2, \cdots, m$). Blocks will be of the form $(\theta_{ip}, \theta_{jq})$ where $p > q$, i.e. pairings or comparisons will be made between groups. For Type A designs, blocks are chosen so as to include *particular* combinations of i and j for *all* combinations of p and q ($p > q$). If all combinations of i and j are included the resulting design is a group divisible design. For Type B designs, blocks are chosen so as to include *all* combinations of i and j for *particular* combinations of p and q ($p > q$). Type C designs will be defined later.

2. Type A designs. Two classes of Type A designs will be discussed, namely designs with $r = (m-1)(n-1)$ and designs with $m = 2$. It is possible to

Received 13 March 1967; revised 6 July 1967.