ON HORVITZ AND THOMPSON'S T_1 CLASS OF LINEAR ESTIMATORS[1]

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1. Introduction. The estimators usually employed to estimate a population mean or total from sample survey data are linear functions of the observations. Horvitz and Thompson (1952) among others have discussed various subclasses of estimators, for example.

(1) Class T_1—estimators of the form $\sum \alpha_r x_r$ where the weight α_r is a function of the order of drawing.

(2) Class T_2—estimators of the form $\sum \beta_i x_i$ where the weight β_i is a function of x_i, the unit drawn.

(3) Class T_3—estimators of the form $\nu_{s_n} \sum x_r$ where the weight ν_{s_n} is a function of the sample taking into account the order of the units.

Obviously, additional classes of estimators can be obtained by associating weights that depend on a combination of these possibilities, (a fact noted by Koop (1957) in his Ph.D. thesis).

Godambe (1955) showed that there does not exist a best (minimum variance unbiased) estimator for the class encompassing the T_2 and T_3 classes.

The present paper discusses the T_1 class estimators as defined in (1) above for the case of sampling with arbitrary probabilities of selection at each draw, and examines some particular sampling designs in the light of this discussion.

2. Notation and terminology. Let X_i $(i = 1, 2, \cdots, N)$ be the value of ith unit under consideration. The problem of present concern is the estimation of

$$T = \sum_{i=1}^{N} X_i \tag{2.1}$$

from a sample of n units.

Let us suppose that the sample is an ordered set—the order being that of the order of draw. Let

$$p_{ir}, \quad i = 1, 2, \cdots, N; \quad r = 1, 2, \cdots, n, \tag{2.2}$$

denote the probability of selecting the ith unit of the population at the rth draw, and

$$P = (p_{ir}) \tag{2.3}$$

be the corresponding $N \times n$ probability matrix in which the rth column denotes

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