NOTE ON DYNKIN'S (α, ξ)-SUBPROCESS OF STANDARD MARKOV PROCESS

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Let α_t be a multiplicative functional of a standard Markov process. E. B. Dynkin [2] has defined "(α, ξ)-subprocess" under certain conditions imposed to α_t. (The conditions are stated as the existence of a suitable stochastic process ξ_t.) In this note, it is shown that (α, ξ)-subprocess exists if and only if α_t is a positive supermartingale of the class (D). For the rigorous proof of this fact, "α_t-additive functional" is introduced and the Meyer decomposition of α_t-additive functional is established.

1. Notations and definitions. Let us first recall the definition of standard Markov process. Let S be a locally compact Hausdorff space with a countable open base and $S^* = S \vee \{\Delta\}$ be the space adjoined Δ to S as an isolated point. $\mathfrak{B}_{S^*}$ is the smallest σ-algebra containing all open sets of S^*. A mapping w; $T = [0, +\infty] \to S^*$ is a *path* if it satisfies (i) $x_t(w) = w_t$ is right continuous, (ii) $x_t(w) = \Delta$ for $t \geqq \zeta(w) = \inf\{t > 0; x_t(w) = \Delta\}(= +\infty$ if $\{\ \} = \emptyset)$ and (iii) $x_t(w)$ has left hand limits in $0 \leqq t < \zeta(w)$. The space of all paths is denoted by W. $\mathfrak{B}_t$ is the smallest σ-algebra on W for which $x_s(w)$ is measurable for $s \leqq t$, and $\mathfrak{B} = \bigvee_{t>0} \mathfrak{B}_t$.

Let P_x, $x \varepsilon S^*$, be a family of probability measures on $(W, \mathfrak{B})$ such that $P_x(B)$, $B \varepsilon \mathfrak{B}$ is $\mathfrak{B}_{S^*}$-measurable and $P_x(x_0(w) = x) = 1$. For a bounded measure μ on $(S^*, \mathfrak{B}_{S^*})$ we define P_μ by $\int \mu(dx)P_x$. A subset N of W which is of P_μ-outer measure 0 for every μ is called a *null set*. The set of all null sets is denoted by $\mathfrak{N}$. $\mathfrak{F}_t$ is the smallest σ-algebra containing $\mathfrak{B}_t$ and $\mathfrak{N}$. Set $\mathfrak{F} = \bigvee_{t>0} \mathfrak{F}_t$. A non-negative $\mathfrak{F}$-measurable function T is a *stopping time* if $\{T \leqq t\} \varepsilon \mathfrak{F}_t$ holds for every $t \geqq 0$. (If $\mathfrak{F}$ and $\mathfrak{F}_t$ are replaced by $\mathfrak{B}$ and $\mathfrak{B}_t$ in the above definition, T is called $(\mathfrak{B})$-*stopping time.*) A stopping time T is called a QHT (*quasi-hitting time*) if (i) $T(\theta_t) + t = T$ for $t \leqq T$ and (ii) $\lim_{t\downarrow 0} T(\theta_t) + t = T$ hold except for a null set, where θ_t is the shift operator defined by $x_s(\theta_t w) = x_{s+t}(w)(\forall_t$, $s \geqq 0)$. For a stopping time T, we define a σ-algebra $\mathfrak{F}_T$ by $\{B \varepsilon \mathfrak{F}$; $B \cap \{T \leqq t\} \varepsilon \mathfrak{F}_t$ for every $t \geqq 0\}$.

$(x_t, \zeta, \mathfrak{F}_t, P_x)$ is called a *standard process* if the following two conditions are satisfied.

(1) (*Strong Markov property*). For each stopping time T,

$$E_x(f(\theta_T w); B) = E_x(E_{x_T}(f); B), \qquad \forall x \varepsilon S^*, \tag{1.1}$$

holds for every bounded $\mathfrak{F}$-measurable function f and $B \varepsilon \mathfrak{F}_T$.

(2) (*Quasi-left continuous before* ζ). For each increasing sequence of stopping

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