OPTIMAL SEQUENTIAL PROCEDURES WHEN MORE THAN ONE STOP IS REQUIRED[1]

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1. Introduction. Let $\{Y_m, m = 1, 2, \cdots\}$ be a (possibly finite) sequence of random variables having a known distribution. These random variables can be observed sequentially, perhaps at some cost, by a statistician who must decide when to stop. If he stops after having observed $Y^{(m)} = (Y_1, \cdots, Y_m)$, he is then presented with an optimal stopping problem that depends on $Y^{(m)}$, i.e., he starts taking observations on another sequence of random variables $\{Y_{mk}, k = m + 1, m + 2, \cdots\}$ and his gain if he stops after observing $Y^{(m,n)} = (Y_1, \cdots Y_m, Y_{m,m+1}, \cdots, Y_{mn})$ is $Z_{mn} = f_{mn}(Y^{(m,n)})$, where f_{mn} is a known real-valued function of all the observations up to that stage. The statistician's problem is to choose a procedure to maximize his expected gain.

This formulation provides a model for studying some extensions of optimal stopping problems that were first considered by Mosteller and Gilbert in [5]. The model is specifically intended to include their two-stop problems (see the examples in Section 3) but can be extended to include their r-stop problems.

The formulation above also applies to some statistical situations in which a preliminary sample can be taken before a sequential decision procedure, or perhaps the design, is decided upon for a second stage. As an example, consider the situation of a man who is going into business for at most 40 years. Suppose that at the end of each year he can choose to continue his operation or he can stop, in which case his net gain is the sum of the profits (perhaps negative) for each of the preceding years. It may be plausible to assume that these yearly profits have a joint distribution that depends on a parameter θ, which in turn can be assumed to have a certain prior distribution. Before starting the business, he may be able to gather information about the value of θ by making observations on random variables (perhaps the profits of similar businesses) at some cost per observation. The problem of maximizing the expected net gain falls under the general formulation above. Other examples are given in Section 3.

2. General solution. The following structure will be assumed throughout: (i) a probability space (Ω, F, P) with points ω; (ii) a non-decreasing sequence $\{F_m, m \geqq 1\}$ of sub-fields of F; (iii) for each fixed $m = 1, 2, \cdots$, a stochastic process $\{Z_{mn}, F_{mn}, n > m\}$ such that $F_m \subset F_{mn} \subset F_{m,n+1} \subset F$ for all $n > m \geqq 1$.

In terms of the informal discussion at the beginning of Section 1, F_m and F_{mn}

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