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NOTE ON A MINIMAX DESIGN FOR CLUSTER SAMPLING

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1. Introduction. The problem of determining the minimax procedure for estimating the population mean, with two-stage cluster sampling has been recently discussed by Aggarwal (1966), without however giving a general solution. A general solution of the problem is presented in this note.

2. Preliminary. We shall refer to Aggarwal's paper (1966) as the 'Main Paper' or shortly as M and use throughout the same notation as in M. It is shown in equation (6.1) in Section 6 of M, that with given m clusters, the minimax sampling scheme is obtained by choosing the n_i, $i = 1, 2, \cdots, m$, so as to minimize the risk,

$$R(\mu, \delta^*) = \{\textstyle\sum_{i=1}^m n_i/(n_i\sigma_b^2 + \sigma_i^2)\}^{-1} + c_b m + \textstyle\sum_{i=1}^m n_i c_i. \tag{1}$$

It is further observed in M, that theoretically speaking the risk (1) should be minimized over the choice of n_i, under the restriction that they be positive integers, but that even without this restriction it does not seem possible to solve the problem of minimizing (1), in general; and that it may be possible only to obtain approximate solutions under some simplifying assumptions. The solution for one such particular case is derived in Section 9 of M.

In the following we obtain a general solution giving non-negative values of n_i, which minimize right hand side of (1), provided the restriction to integral values is ignored.

3. Main result. We put,

$$S = \textstyle\sum_{i=1}^m n_i/(n_i\sigma_b^2 + \sigma_i^2), \tag{2}$$

$$R = R(\mu, \delta^*). \tag{3}$$

We are concerned with only the positive quadrant of the m-spcce of the variables n_i, $i = 1, 2, \cdots, m$, defined by $n_i \geqq 0$. We shall refer to this space as the n_i-space. Suppose R has a minimum (in the calculus sense) in the positive quadrant of the n_i-space. Then at the minimum point we must have

$$\partial R/\partial n_i = 0, \qquad i = 1, 2, \cdots, m.$$

By differentiation, we obtain from (1)

$$\partial R/\partial n_i = -\sigma i^2/S^2(n_i\sigma_b^2 + \sigma_i^2)^2 + c_i, \tag{4}$$

so that by equating $\partial R/\partial n_i$ to 0, we have

$$n_i\sigma_b^2 + \sigma i^2 = \sigma i/Sc_i^{\frac{1}{2}}, \tag{5}$$

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