The Annals of Mathematical Statistics
1968, Vol. 39, No. 1, 233–241

THE CALCULATION OF DISTRIBUTIONS OF KOLMOGOROV-SMIRNOV TYPE STATISTICS INCLUDING A TABLE OF SIGNIFICANCE POINTS FOR A PARTICULAR CASE

BY MARC NOÉ AND GEORGES VANDEWIELE

M.B.L.E. Research Laboratory, Brussels and Free University of Brussels

1. Introduction. Let $X_1 \leqq X_2 \leqq \cdots \leqq X_n$ be the order statistics of a sample of size n from a continuous distribution function $F(x)$ and let $F_n(x)$ be the corresponding empirical distribution function. Let $G(x)$ and $H(x)$ be two functions of x. We will consider the probabilities

$$\bar{P}_n = P\{\inf_x [G(x) - F_n(x)] \geqq 0\}, \tag{1}$$

$$\underline{P}_n = P\{\inf_x [F_n(x) - H(x)] \geqq 0\};$$

$$P_n = P\{\inf_x [G(x) - F_n(x)] \geqq 0, \ \inf_x [F_n(x) - H(x)] \geqq 0\}. \tag{2}$$

These probabilities are related to the statistics of the Kolmogorov-Smirnov type in the following way: The corresponding one-sided statistic has the distribution function

$$P\{\sup_x m^{\frac{1}{2}}[F_n(x) - F(x)]\psi[F(x)] \leqq \lambda\} \tag{3}$$

which is a probability of the form (1). The two-sided statistic has the distribution

$$P\{\sup_x n^{\frac{1}{2}}|F_n(x) - F(x)|\psi[F(x)] \leqq \lambda\} \tag{4}$$

which is a special case of (2). In these expressions $\psi(x)$ is a (non-negative) weight function. A discussion of these statistics can be found e.g. in Kendall and Stuart [4].

Wald and Wolfowitz [8] [9] have given recursion formulas for computing $\bar{P}_n$, $\underline{P}_n$ and P_n. Daniels [2] was led to a probability of the same form as $\bar{P}_n$ or $\underline{P}_n$ in connection with a study of the strength of bundles of threads. He found recursions slightly more general than in [8] by a very similar method. In Section 2 we give a simple derivation of still more general formulas for $\bar{P}_n$ and $\underline{P}_n$, which contain a wider choice of recursions, so that the numerical computability can be taken into account. Note that two non-recursive formulas for $\bar{P}_n$ or $\underline{P}_n$ are given by Daniels [2], but unfortunately they are not easily tractable.

In Section 3 a formula for P_n is derived which is simpler than the corresponding formula of [8] but is valid only under certain conditions. Furthermore in Sections 2 and 3 bounds of $\bar{P}_n$, $\underline{P}_n$ and P_n are given with a view to approximate calculations.

Received 1 June 1967; revised 11, July 1967.