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TOWARDS A THEORY OF GENERALIZED BAYES TESTS[1]

BY R. H. FARRELL

Cornell University

1. Introduction. The main result of this paper is Theorem 7.1. Stated there is a necessary and sufficient condition for the admissibility of tests in the exponential case when the hypothesis set H_0 is compact and "topologically" separated from the alternative H_1. In the theorem we ask that the entire parameter space Ω be a closed convex cone in Euclidean k-space $\mathbb{R}_k$. The proof that tests satisfying the stated condition are admissible is relatively easy and is almost a direct consequence of an admissibility theorem for generalized Bayes tests proven in Section 5. The proof that the stated condition is necessary is much harder and requires the results of Section 2, Section 3, Parts of Section 4, and a lengthy argument in Section 7.

This paper originated out of efforts on the author's part to see what could be done with a theory of generalized Bayes tests. This can be considered to be a continuation of work begun in Farrell [5] in which some complete class theorems in estimation problems were obtained.

It was discovered that in the case of Birnbaum's [2] necessary and sufficient condition all admissible tests are generalized Bayes tests (see Theorem 4.1) and conversely under much less restricted conditions generalized Bayes tests are admissible (see Theorem 5.1 and 5.2). L. D. Brown has given the author several examples presented in Section 6. In Example 6.1 the hypothesis set H_0 is a compact convex set and $H_1 = \Omega - H_0$. The probabilities form an exponential family (see below) and *every* test function is admissible. This example completely destroys the hope of completely describing admissible tests in the exponential case by generalized Bayes procedures.

Example 6.2 of L. D. Brown led the author to Theorem 7.1. In this example H_0 contains two points x_1, x_2, (hence is compact) and H_1 may be considered to be any closed subset of Ω disjoint from H_0 so long as H_1 contains $(x_1 + x_2)/2$ and a sequence of points (x_n, y_n) with $\lim_{n\to\infty} y_n = \infty$. An admissible test is described that is not a generalized Bayes test but within a certain subfamily of tests is in fact a Bayes test. The necessary and sufficient condition stated in Theorem 7.1 has to do with choice of the right subfamily of tests.

Throughout subsequent sections the parameter space will be denoted by Ω, the hypothesis set by H_0 and the alternative set by H_1. We assume $H_0 \cap H_1 =$ null set, but allow $H_0 \cup H_1$ to be a proper subset of Ω.

$\{f_\omega(\cdot), \omega \varepsilon \Omega\}$ will be a family of generalized density functions on a set X,

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