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HYPER-ADMISSIBILITY AND OPTIMUM ESTIMATORS FOR SAMPLING FINITE POPULATIONS[1]

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1. Introduction. The technique of varying probability sampling was partially generalized by Horvitz and Thompson (1952), who furnished an unbiased estimator of the population total. Their estimator, which we shall call the H.T. estimator for brevity, is in fact applicable in a very general set-up and turned out to be an admissible estimator. Godambe (1955) generalized the concepts of sampling design and linear estimators. The important negative result that emerged from his investigation is the nonexistence of a uniformly minimum variance (umv, for brevity) estimator among homogeneous linear unbiased estimators of the population total. While Godambe supposed this to be true for all sampling designs, there are nontrivial exceptions to this result which are completely characterized by the author (1966a) as 'uni-cluster designs.'[3] These, however, have the serious drawback that unbiased variance estimators do not exist for them. Barring therefore these unicluster designs, we have only some negative results, of which Basu's (1958) result concerning the inadmissibility of estimators that depend on the order or repetitions of units in a sample, is the important one. Koop[4] (1957) and Prabhu Ajgaoukar (1962) proved the nonexistence of a umvue even in certain subclasses of linear unbiased estimators. Roy and Chakravorty (1962) proposed the additional criterion of linear invariance but even this did not give a umvue. Godambe (1955) and Hájek (1959) followed the Bayesian approach to the problem and obtained "best strategies" for some important practical situations when a particular type of auxiliary information is available. In all other situations the problem of an 'optimum' estimator still remains unsolved.

In Section 3 of this paper we propose a new criterion which we name *hyper-admissibility* (*h*-admissibility for short). This criterion gave a unique optimum estimator which is the H.T. estimator, in a very wide class of unbiased estimators and for all non-unicluster designs. Even for unicluster designs the H.T. estimator

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[3] In his later paper (1965) Godambe noted this exception to his earlier result.

[4] Koop introduced seven classes of homogeneous linear estimators, identified Godambe's class as his T_5-class and claims that his T_7-class is wider than Godambe's class. However, recalling that Godambe defines his sample as an *ordered* sequence, it can be seen that the two classes are, in fact, identical.