*The Annals of Mathematical Statistics*
1968, Vol. 39, No. 2, 606–620

# ADMISSIBILITY OF THE SAMPLE MEAN AS ESTIMATE OF THE MEAN OF A FINITE POPULATION

By V. M. Joshi

*Maharashtra Government, Bombay*

**1. Introduction.** In previous papers, the sample mean (1965-II), and later a ratio estimate (1965-IV, Section 5), were shown, with the squared error as the loss function, to be admissible as estimates of the population mean, whatever be the sampling design. The validity of these results was however restricted by the assumption of one particular loss function, thus raising the question whether these estimates remain admissible for other, equally valid, loss functions. In this paper, the restriction on the loss function is removed and the admissibility of the ratio estimate, (which includes the sample mean as a particular case) is shown to hold generally for any loss function, which satisfies certain mild conditions, which would be satisfied by almost any loss function assumed in practice.

**2. Notation and definitions.** $U$ denotes the population consisting of units $u_1, u_2, \cdots, u_N$; with unit $u_i$, is associated a variate value $x_i$, $i = 1, 2, \cdots, N$; $x = (x_1, x_2, \cdots, x_N)$ denotes a point in the sample space $R_N$; a sample $s$ denotes a subset of $U$; $S$ denotes the set of all possible samples $s$; a probability function $p$ is defined on $S$, such that

$$p(s) \geqq 0 \quad \text{for all} \quad s, \quad \text{and} \quad \sum_{s \varepsilon S} p(s) = 1.$$

Following Godambe and Joshi (1965-I), the pair $(S, p)$ is called the sampling design. A sample $s$ is drawn from $S$ according to $p$. Then we define,

Definition 2.1. An estimate $e(s, x)$ is a real function $e$ defined on $S \times R_N$, which depends on $x$, through only those $x_i$ for which $u_i \varepsilon s$.

The above definitions of sampling design and estimate are wide enough to cover all sampling procedures and classes of estimates; for a brief account we refer to Godambe and Joshi (1965-I), Section 5.

Let $V(t)$ be the loss function, where $t$ is the absolute value of the difference between the estimate and the true value. We assume that $V(t)$ is non-decreasing and that it satisfies one more condition, which for convenience will be formulated later (in (29)).

Let $T_N$ be the population mean, i.e.

$$T_N(x) = N^{-1} \sum_{i=1}^{N} x_i. \tag{1}$$

With the loss function $V(t)$, we define admissibility of estimates of $T_N(x)$. For a given sampling design $d$, let $\bar{S}$ be the subset of $S$, consisting of all those samples for which $p(s) > 0$. Then,

Received 15 February 1967; revised 17 October 1967.