The Annals of Mathematical Statistics
1968, Vol. 39, No. 2, 567–582

JACKKNIFING VARIANCES[1]

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1. Introduction. The Tukey jackknife [18], [19], [11], which is an extension of an idea of Quenouille [12], is a rough-and-ready statistical tool which (a) reduces bias and (b) produces approximate confidence intervals. It exactly eliminates a $1/n$ bias term. Its approximate confidence intervals are a godsend in problems where messy distribution theory prohibits the formation of exact confidence limits.

It has been demonstrated that the jackknife can be beneficially applied in ratio problems (Quenouille [13], Durbin [6], Rao [14], Rao and Webster [15], Deming [5]), in maximum likelihood estimation (Brillinger [3]), and in transformations of statistics (Miller [9]). A recent proposed application is the construction of confidence limits for estimates of parameters in a functional relationship (Brillinger [4]).

Indiscriminate universal application of the jackknife can be hazardous. This is illustrated in the case of interval estimation for a truncation point (Miller [9]), although under restrictions on the probability density the jackknife still performs satisfactorily (Robson and Whitlock [16]). Lincoln Moses in unpublished work has shown that the jackknife runs into trouble for interval estimation on the median.

The purpose of this paper is to examine how the jackknife performs in testing hypotheses on variances. It is well known for this problem that it is disastrous to base a test on the χ^2 or F distribution because of extreme sensitivity of the distribution to nonnormality. A variety of alternatives to the classical techniques have been proposed. Some of these involve arbitrarily dividing the data into groups. As soon as the idea of division into groups creeps forth, the jackknife cries out to be tested.

Two objectives are accomplished in this paper: (1) Another technique is added to the short list of tests which are robust and reasonably powerful for testing variances. (2) Another problem is recorded in which the jackknife performs admirably so statisticians should be imbued with courage to try the jackknife elsewhere.

Section 2 contains a description of the jackknife technique. The asymptotic distribution theory is worked out in Section 3. In Section 4 the jackknife is compared with other techniques for testing variances, both for large samples and for small samples. Section 5 closes the paper with a discussion of the performance of the jackknife with regard to testing variances and in general. The Appendix contains a proof that Levene's z test is not asymptotically distribution-free.

Received 9 June 1967.
[1] Research supported in part by Public Health Service Grant USPHS-5T1 GM 25-09.