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A REMARK ON HITTING PLACES FOR TRANSIENT STABLE PROCESS[1]

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1. Introduction. In this note we will consider a non-degenerate, drift free, d-dimensional stable process $X(t)$, having exponent α, $0 < \alpha \leqq 2$, and transition density $p(t, x)$ satisfying the scaling property

$$p(rt, r^{1/\alpha}x)r^{d/\alpha} = p(t, x). \tag{1.1}$$

Thus if $\alpha = 1$ the process must be isotropic, while if $\alpha \neq 1$ the process is arbitrary. In addition, we assume that $X(t)$ is a version of the process which is a standard Markov process. (See [1] for a description of a standard process.)

In [3] Taylor states the following:

THEOREM 1. *Assume* $p(1, 0) > 0$. *Then* $p(t, x) > 0$ *for all* $t > 0$ *and all* $x \varepsilon R^d$ (*d-dimensional Euclidean space*).

The proof given by Taylor seems incomplete. Our first result will be to present a complete proof of this useful fact. It is a pleasure for me to thank J. Folkman for helpful conversations on this matter. In fact, by using induction on the dimension, and a much more refined version of the arguments used to prove this theorem, Folkman was able to demonstrate the following much more precise fact:

$$\{y : p(t, y) > 0 \quad \text{for some} \quad t\} = \{y : p(t, y) > 0 \quad \text{for all} \quad t > 0\}.$$

Suppose now, in addition to the assumptions made above, that $\alpha < d$ so that $X(t)$ is a transient process.

Let B be a bounded Borel subset of R^d, and let $T_B = \inf\{t > 0 : X(t) \varepsilon B\}$ ($= \infty$ if $X(t) \not\varepsilon B$ for all $t > 0$) denote the first hitting time of B. Recall that B is said to be polar if $P_x(T_B < \infty) \equiv 0$. Let $\hat{T}_B$ denote the quantity T_B for the dual process $-X(t)$. The potential kernel of the process $X(t)$ is the quantity

$$g(x) = \int_0^\infty p(t, x)\, dt,$$

while the kernel for the dual process is $\hat{g}(x) = g(-x)$. For an event A, let $P_x(A)$ denote the probability of A given $X(0) = x$, and for a random variable Z, let $E_x[Z; A] = \int_A Z(w)P_x(dw)$.

The main purpose of this note is to establish the following:

THEOREM 2. *Suppose* $\alpha < d$ *and that* $p(1, 0) > 0$. *Let* B *be a bounded Borel set, and let* f *be an arbitrary continuous function on the closure* $\bar{B}$ *of* B. *If the kernel* $g(x) < \infty$, $|x| = 1$, *and is such that for any compact* A,

$$\lim_{|x|\to\infty} \sup_{y \varepsilon A} g(x + y)g(x)^{-1} = 1, \tag{1.2}$$

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