The Annals of Mathematical Statistics
1968, Vol. 39, No. 2, 347–357

SOME RESULTS ON MULTITYPE CONTINUOUS TIME MARKOV BRANCHING PROCESSES[1]

By Krishna Balasundaram Athreya

Stanford University

1. Introduction. Of late there has been a lot of interest in multitype continuous time Markov branching processes(MCMBP). It was recently noted that there is a fundamental and yet simple connection between classical urn schemes like Polya's and Friedman's, etc. and multitype continuous time Markov branching processes (see [1]). Problems on urn schemes have their counterparts in branching processes and it is while attempting to solve these that the author felt the need for a systematic study of the MCMBP. The present paper is a partial answer to this need (see also [1], [2], [3]).

In this paper we develop in a systematic way some basic properties of multitype continuous time Markov branching process. Although a few of these properties are elementary and not entirely new in content we present them here for the sake of completeness. But there are two very important results which we believe are new. We prove them under minimal assumptions. (See Section 2 for notations and preliminaries.) Let $\{X(t); t \geqq 0\}$ be a MCMBP and let A be the infinitesimal generator of the mean matrix semigroup $\{M(t); t \geqq 0\}$ where $M(t) = ((m_{ij}(t)))$ and $m_{ij}(t) = ((E(X_j(t) \mid X_r(0) = \delta_{ri}, r = 1, 2, \cdots, k)))$ and δ_{ij} are Kronecker deltas. Assuming positive regularity and nonsingularity of the process we establish the following:

Theorem 1. *Needing nothing more than the existence of the first moments, we have*

$$\lim_{t\to\infty} X(t, \omega)e^{-\lambda_1 t} = W(\omega)u \qquad \textit{exists} \qquad \text{wp } 1$$

where $W(\omega)$ is a nonnegative numerical valued random variable, λ_1 is the maximal real eigenvalue of A and u is an appropriately normalized vector satisfying $u^*A = \lambda_1 u^*$ where u^* is the transpose of u.

Theorem 4. *Let $\lambda_1 > 0$ so that $P\{X(t) = 0 \text{ for some } t\} < 1$ for any nontrivial makeup and assume second moments exist. Then*

$$\lim_{t\to\infty} P\{0 < x_1 \leqq W \leqq x_2 < \infty, (v \cdot X(t) - e^{\lambda_1 t}W)(v \cdot X(t))^{-\frac{1}{2}} \leqq y\}$$
$$= P\{0 < x_1 \leqq W \leqq x_2 < \infty\}\Phi(y/\sigma)$$

where v is an appropriately normalized vector satisfying $Av = \lambda_1 v$, σ^2 an appropriate constant and $\Phi(x)$ is the normal distribution function.

Here is an outline of the rest of the paper. In Section 2 we describe our set up and construct some martingales. Section 3 establishes Theorem 1. Assuming the

Received 18 October 1967.

[1] Research supported in part under contracts N0014-67-A-0112-0015 and NIH USPHS 10452 at Stanford University.