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SOME RULES FOR A COMBINATORIAL METHOD FOR MULTIPLE PRODUCTS OF GENERALIZED k-STATISTICS

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1. Introduction. Dwyer and Tracy [2] gave some rules which are useful in obtaining formulae for products of two generalized k-statistics in terms of linear combinations of such statistics. These rules included generalizations of certain rules of Fisher [3] and Kendall [4], Wishart [8], and Tukey [7]. All the rules in [2] can be generalized to give rules applicable when forming products of more than two generalized k-statistics. The rules concern determination of pattern functions associated with various patterns. This paper indicates a generalization of the four such rules in [2] and establishes four additional rules.

2. General notation and background material. A π-part partition of a positive integer p, denoted by P as in [2], may be represented by

$$P = p_1^{\pi_1} p_2^{\pi_2} \cdots p_s^{\pi_s} \tag{2.1}$$

with the convention $p_1 > p_2 > \cdots > p_s > 0$, where the order of P is the number of parts $\pi = \sum_{i=1}^{s} \pi_i$ and where $\sum_{i=1}^{s} p_i \pi_i = p$. It may also be written in the form

$$P = p_1 p_2 \cdots p_\pi \tag{2.1$'$}$$

with the convention $p_1 \geqq p_2 \geqq \cdots \geqq p_\pi > 0$. Here p itself may be considered as a 1-part partition of p. We call p_i a proper part of p if $p_i < p$.

The augmented symmetric function of the sample values $x_1, x_2, \cdots, x_n$ is given by [4], p. 276,

$$\sum x_i^{p_1} x_j^{p_1} \cdots x_q^{p_2} x_r^{p_2} \cdots x_u^{p_s} x_v^{p_s} \cdots = [p_1^{\pi_1} p_2^{\pi_2} \cdots p_s^{\pi_s}] = [p_1 p_2 \cdots p_\pi] = [P] \tag{2.2}$$

and the average augmented symmetric function (sample), which Tukey [7], p. 38, calls the symmetric mean or bracket $\langle p_1 p_1 \cdots p_\pi \rangle$, may be written as

$$m_P' = \langle P \rangle = [P]/n^{(\pi)} \tag{2.3}$$

and hence [4], p. 276, [1], p. 42,

$$E(m_P') = \mu_P' = \mu'_{p_1} \mu'_{p_2} \cdots \mu'_{p_\pi} \tag{2.4}$$

where μ''s are the moments about the origin.

The partition coefficient $C(P)$, as defined in [2], is the number of ways that the distinct units of p may be combined into sets of indistinguishable parcels de-

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