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BAYES SEQUENTIAL DESIGN OF FRACTIONAL FACTORIAL EXPERIMENTS FOR THE ESTIMATION OF A SUBGROUP OF PRE-ASSIGNED PARAMETERS[1]

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0. Introduction. The objective of the present study is to investigate and find optimal sequential designs of fractional factorial experiments, for the purpose of estimating a pre-assigned subgroup of parameters. We are given a factorial system of $N = 2^m$ treatment combinations. Without loss of generality, assume that the pre-assigned subgroup of parameters consists of the first $S = 2^s (s < m)$ main effects and interactions. (The parameters are arranged according to the standard order, see Ehrenfeld and Zacks [2].) All the other $K = N - S$ parameters are considered as nuisance parameters. It is desired to estimate the subvector of S pre-assigned parameters. Randomized unbiased procedures were studies by Ehrenfeld and Zacks [5], [6], [13] in which $n (1 \leqq n \leqq M = 2^{m-s})$ blocks consisting of S treatment combinations are chosen. The design of these n fractional replicates is carried according to those procedures by a non-sequential manner. The question is whether one can improve and reduce total estimation risk by designing the fractional replicates sequentially, and after each stage of experimentation adjusting the appropriate estimator according to the information obtained concerning the parameters (pre-assigned as well as nuisance). It is well known that the best truncated sequential procedure for estimating the mean of a normal distribution with a known variance, $\mathfrak{N}(\alpha, 1)$ say, when the loss function is quadratic is a fixed sample procedure (see Hodges and Lehmann [8], and J. Wolfowitz [12]). In the present model we have observations following an S-variate normal distribution with a known covariance matrix $\sigma^2 I_S$. The mean vector is $A(\alpha, \beta)'$, where A is an $S \times N$ matrix; α is an S-dimensional vector to be estimated, and β is a K-dimensional vector of nuisance parameters. The question is whether under this model there exists a sequential procedure which is better than any fixed sample procedure. The present study is devoted to the solution of this problem. We search for a Bayes sequential design procedure, when the a-priori distribution of the parameters is assumed to be normal and the loss function is quadratic. As expected, the result is that the Bayes procedure for a prior normal distribution is of a fixed sample size. The optimal number of fractional replicates to perform is a function of the cost of experimentation and the prior dispersion matrix of the parameters. The main part of the investigation is then devoted to the problem of the best choice of n fractional replicates out of the $M = 2^{m-s}$ possible ones. It is proven that the Bayes procedure is to choose n different blocks

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