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INFERENCE PROBLEMS ABOUT PARAMETERS WHICH ARE SUBJECTED TO CHANGES OVER TIME[1]

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Introduction. Consider a situation where a random variable is observed sequentially over time and the distribution of this random variable is subject to a possible change at every point in the sequence. We discuss some problems connected with this situation.

In the first two sections, we assume that the change is random in nature and affects the mean of the distribution. The study of this problem is centered about a model introduced by Chernoff and Zacks [1]. We first consider the problem of estimating the current value of the mean on the basis of a set of observations taken up to present. The problem has been treated in some detail before [1] but it has been assumed throughout implicitly that certain parameters occurring in the model are known. In Section one, we derive a procedure for estimating the current value of the mean on the basis of a set of observations taken at successive time points when nothing is known about the other parameters occurring in the model.

Section two considers another important aspect of the problem, namely, to estimate the various points of change. We handle the problem in the framework of empirical Bayes procedure and use an idea similar to that of Tainiter [6] to derive a sequence of tests to be applied at each stage. This sequence of tests will be shown to be "asymptotically reasonable" in a certain sense.

In Section three, we consider n independent observations of a random variable taken at successive time points. It is further assumed that the distribution of the random variable belongs to the one parameter exponential family. We examine the problem of testing the equality of these n parameters against the alternative that the parameter has changed r-times at some unknown points where r is some finite positive integer less than n. A test procedure is obtained by generalizing the techniques used by Kander and Zacks [2] who studied the case $r = 1$. Under quite general conditions, the distribution of the test statistic is shown to be asymptotically normal both under the null and the alternative hypotheses. This kind of problem has also been studied by Page [5].

1. Estimation of the present value of the mean. Let $\mathbf{x}' = (x_1, x_2, \cdots, x_n)$ be the observation vector where the observations are taken at n successive time points. Then following Chernoff and Zacks [1], the co-ordinates of $\mathbf{x}$ satisfy the

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