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ESTIMATION OF STOCHASTIC SYSTEMS: ARBITRARY SYSTEM PROCESS WITH ADDITIVE WHITE NOISE OBSERVATION ERRORS[1]

By G. Kallianpur and C. Striebel

University of Minnesota

1. Introduction. The principal result of this paper, stated in Theorem 3, is a form of the Bayes theorem which is required for the solution of many problems in the control and estimation of stochastic systems. Although the original motivation for the problem treated here is in the field of control, it is more convenient to formulate it in terms of estimation. Its application to control will be discussed in a later paper.

We shall be concerned with the estimation of a "system process" $x(t)$, $0 \leqq t \leqq T$ which we assume to be defined as a stochastic process $x(t, \eta)$ on a known probability space $(\Omega_X, \mathcal{B}_X, P_X)$, $(\eta \in \Omega_X)$. It is further assumed that the system process cannot be observed directly. Instead we have available an "observation process" $z(\tau)$ which is given by

$$z(\tau) = \int_0^\tau x(u)\,du + w(\tau), \qquad 0 \leqq \tau \leqq T, \tag{1.1}$$

where $w(\tau)$ is a standard Wiener process independent of the system process. Our available data is $z(\tau)$, $0 \leqq \tau \leqq t$, for t fixed in the interval $0 \leqq t \leqq T$, and using this data we wish to estimate some functional of the system process $x(\tau)$, $0 \leqq \tau \leqq T$,

$$G[x(\tau, \eta); 0 \leqq \tau \leqq T]. \tag{1.2}$$

It will be assumed that the resulting function $g(\eta)$ defined on $(\Omega_X, \mathcal{B}_X, P_X)$ by

$$g(\eta) = G[x(\tau, \eta); 0 \leqq \tau \leqq T] \tag{1.3}$$

is integrable.

The system process, or more precisely, the space Ω_X on which it is defined corresponds to the parameter space in the usual Bayes approach to the theory of estimation. Thus the probability P_X is the *a priori* distribution for the unknown parameter; the process $z(\tau)$, $0 \leqq \tau \leqq t$, is the observed random variable and we wish to estimate the function $g(\eta)$ defined on the parameter space.

We shall assume a squared error loss function. Hence we wish to find an estimate $\delta(z(\tau), 0 \leqq \tau \leqq t)$ which minimizes

$$E(g - \delta)^2. \tag{1.4}$$

To the memory of Norbert Wiener.

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