The Annals of Mathematical Statistics
1968, Vol. 39, No. 5, 1540-1548

CONSTRUCTION OF ROOM SQUARES

BY R. G. STANTON AND R. C. MULLIN

University of Waterloo

1. Introduction. The particular design which has since become known as a Room square or a Room design was first introduced by T. G. Room in a brief note [4]. We shall introduce a notation slightly different from that used by Room. We take the $2n$ symbols $1, 2, 3, \cdots, 2n - 1, \infty$. Then a Room design of order $2n$ consists of a square of side $2n - 1$ with each compartment of the square either being blank or containing an unordered pair of the symbols $1, 2, 3, \cdots, 2n - 1, \infty$. Furthermore, each row and each column of the square contains $n - 1$ empty cells and n cells containing a pair of symbols; the totality of symbols appearing in each row and in each column is just the total number, $2n$, of symbols. We further require that each of the $n(2n - 1)$ possible distinct pairs of symbols shall occur exactly once in a cell of the square.

We shall also find it useful to refer to a Room design of order $2n$ as a Room square of side $2n - 1$. Room squares may thus exist only for odd integers $k = 2n - 1$.

Room's initial note pointed out that a Room square existed trivially for $k = 1$, did not exist for $k = 3$ or 5, did exist for $k = 7$. Archbold and Johnson [2] gave a construction for $k = 7, 31, 127, \cdots$, that is, for any k which is less by unity than an odd power of 2; they also pointed out the applications of Room squares in statistical design and sketched the appropriate analysis of variance for such a design. Archbold [1] gave a different construction, based on difference sets, which produced squares of side $k = 7, 11, 19, 23$; again this method failed for $k = 15$, just as the earlier method had. Bruck [3] pointed out the connection between Room squares and quasigroups, gave an elegant new construction for the squares given by Archbold and Johnson [2], and proved that if squares of sides a and b existed, then one could use a type of Kronecker product to get a square of side ab; in particular, this gave a square of side 49. Finally, Weisner [5] constructed a square of side 9.

The preceding background sketch shows that relatively few Room squares are known. For instance, the only values of k less than 100 for which squares have been constructed are $k = 7, 9, 11, 19, 23, 31, 49, 63, 77, 81, 99$. It is the aim of the present article to outline a method whereby it appears that Room squares of any side k (k odd, $k > 5$) may be constructed; the actual construction has been carried out for all odd numbers k from 7 to 47 inclusive. Empirical evidence suggests that there exists a very large number of Room squares of given side k.

2. Construction of a cyclic Room square of side 11. We shall illustrate the method employed by discussing the case $k = 11$ in detail. We shall restrict ourselves to the construction of cyclic Room squares. These are squares in which the

Received 16 January 1967.