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ON THE ERGODICITY FOR NON-STATIONARY MULTIPLE MARKOV PROCESSES

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0. Introduction. In recent years, among many problems concerning non-stationary Markov processes with discrete time, much attention has been drawn to the problem of ergodicity. The problem is treated by Hajnal [3], Mott [6], Kozniewska [5], in the case of simple (i.e. first order) Markov processes with finite state space, the state space being the same for every instant t. Recently, one of the authors, in collaboration with Dorel [1], has extended the results of [5] to the case where the state space, for each t, is any measurable space. Recently also, Iosifescu [4], studied the uniform ergodicity of non-stationary multiple Markov processes. Using a definition of Iosifescu, we extend the results of [1] to kth order processes, $k \geqq 1$.

In the course of the present investigation, we introduce the notion of ergodicity of power h, h being a positive integer, and show that under certain conditions of uniformity, every non-stationary Markov process of order k which is ergodic of power h, with $h \geqq k$, is ergodic in the sense defined by Iosifescu. This simplifies considerably the eventual verification of ergodicity of a process (in the sense of Iosifescu). It is also shown, by means of examples, that there exist kth order Markov processes, ergodic of power h, with $h < k$, which are not ergodic in the sense of Iosifescu, thus justifying the notion of ergodicity of power h. The use of the "associated" simple process corresponding to the kth order process simplifies many proofs and, moreover, in the finite case, allows the passage from certain rectangular matrices to square matrices, thus facilitating the computations and the practical verification of the results. This article also shows that there are problems concerning kth order Markov processes which cannot be reduced to problems concerning first order Markov processes (for example, ergodicity of power $0 < h < k$).

1. Definitions, notation and preliminary remarks. In the following, we shall denote by $\mathbf{N}$ the set of non-negative integers, $\mathbf{N}^*$ the set of positive integers, and $\mathbf{1}_B$ the indicator of the set B.

Let $(\mathfrak{X}', \mathfrak{B}')$ and $(\mathfrak{X}'', \mathfrak{B}'')$ be two measurable spaces. We recall that a transition probability P from $(\mathfrak{X}', \mathfrak{B}')$ to $(\mathfrak{X}'', \mathfrak{B}'')$ is a mapping $P : \mathfrak{X}' \times \mathfrak{B}'' \rightarrow [0, 1]$ such that $\forall\, x \in \mathfrak{X}'$, $P(x, \cdot)$ is a probability measure on $\mathfrak{B}''$, and $\forall\, B \in \mathfrak{B}''$, $P(\cdot, B)$ is a real random variable defined on $(\mathfrak{X}', \mathfrak{B}')$.

Let $((\mathfrak{X}_t, \mathfrak{B}_t))_{t \in \mathbf{N}^*}$ be a sequence of measurable spaces. We shall denote the product σ-algebra of $(\mathfrak{B}_t)$, $1 \leqq t \leqq n$, (resp. $t \in \mathbf{N}^*$), by $\times_{t=1}^{n} \mathfrak{B}_t$ (resp. by $\times_{t \in \mathbf{N}^*} \mathfrak{B}_t$). In the particular case when $\mathfrak{X}_t = \mathfrak{X}$, we let $\times_{t=1}^{n} \mathfrak{B}_t = \times^n \mathfrak{B}$.

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