The Annals of Mathematical Statistics
1968, Vol. 39, No. 5, 1391–1401

ON A THEOREM BY DOBRUSHIN

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1. Introduction. In 1956 Dobrushin obtained in [2] an interesting theorem on the asymptotic convergence to the Poisson process of a randomly translated one-dimensional infinite particle system. Within the context of his Theorem 1 a necessary and sufficient condition for convergence was obtained.

It was pointed out by J. Goldman that Dobrushin's sufficiency proof was wrong, a serious error occuring in equation (17) of [2]. It turns out also that equation (22) used in giving a trivial proof of the necessary condition is wrong, and the necessary condition is incorrect as stated in the theorem.

In this paper we will correct these two errors of Dobrushin and also generalize the results to d-dimensions. Also we will consider analogous results when the sets may possess boundaries having positive measure.

Finally, we will give examples where the conditions are satisfied. These examples which have been treated in special cases by other authors, involve renewal theory, random walks, and processes with independent increments, and processes with random constant velocities.

In addition to the cited work of Dobrushin, related work has been done by Doob [3], Marayuma [6], Watanabe [10], Lamperti [5], Breiman [1], Thedeen [8], Goldman [4], and Warnshuis [9].

2. Definitions and statements of results. Let X denote a d-dimensional closed subgroup of R^d. With no loss of generality we can assume that X is of the form

$$X = \{x = (x^1, \cdots, x^d) \mid x^k \text{ are integers for } d_1 < k \leqq d\}.$$

Set $Z^d = \{x \mid x^k$ are integers for $1 \leqq k \leqq d\}$. If $d_1 = d$, then $X = R^d$; and if $d_1 = 0$, then $X = Z^d$.

Set $\Delta_m = \{x \varepsilon X \mid 0 \leqq x^k < m$ for $1 \leqq k \leqq d\}$ and set $\Delta = \Delta_1$. Set $Z_m{}^d = Z^d \cap \Delta_m$. Finally, set

$$U = \{u \varepsilon Z \mid u^k = 1 \quad \text{for some } k, \text{ and } u^j = 0 \quad \text{for } j \neq k\}.$$

Then U consists of d "unit vectors." For $0 \leqq a < \infty$ and $x \varepsilon X$, set $a \odot x = (ax^1, \cdots, ax^{d_1}, x^{d_1+1}, \cdots, x^d)$.

Let $| \ |$ denote Haar measure on X, defined as the product of Lebesgue measure on the first d_1 coordinates of X and counting measure on the last $d - d_1$ coordinates. Let $\mathcal{B}$ denote the collection of all relatively compact Borel subsets of X. All subsets of X considered later on will be members of $\mathcal{B}$. Let $\mathcal{A}$ denote the subcollection of sets $A \subset \mathcal{B}$ such that $|\partial A| = 0$.

Let the time parameter set T be given either as $T = \{0, 1, 2, \cdots\}$ or

Received 21 August 1967.
[1] Research supported by NSF Grant GP8049.