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AN EXTENSION OF PAULSON'S SELECTION PROCEDURE

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1. Introduction. Suppose that $\pi_1, \pi_2, \cdots, \pi_k$ are k populations in which we may observe the independent random variables $X_1, X_2, \cdots, X_k$, respectively. We assume that the distribution of X_i is a member of the one-dimensional Koopman-Darmois family with density $\exp\{P(x)Q(\theta_i) + R(x) + S(\theta_i)\}$, $i = 1, 2, \cdots, k$. Let $\tau = Q(\theta)$ and suppose that the ordered set of the τ-values of $\pi_1, \pi_2, \cdots, \pi_k$ are denoted by $\tau_{[1]} \leqq \tau_{[2]} \leqq \cdots \leqq \tau_{[k]}$. These τ-values are assumed to be unknown, and if $\tau_{[k]} > \tau_{[k-1]}$, we refer to the population associated with $\tau_{[k]}$ as the best population. In this paper we obtain a sequential procedure which guarantees that the probability of selecting the best population is at least a specified amount whenever $\tau_{[k]}$ exceeds $\tau_{[k-1]}$ by a specified quantity. This procedure is an extension of Paulson's [4] procedure for selecting the normal population with the greatest mean. The major difference between the two procedures is that Paulson's is truncated while the one obtained here is not.

An exhaustive discussion of the different aspects of the problem considered in this paper will be found in the monograph by Bechhofer, Kiefer and Sobel [1], who have given a different sequential procedure which guarantees the same probability requirements. The notations used in the present paper largely follow those used in the monograph. The procedure obtained in this paper solves the ranking problem when the measure of distance between two populations π_i and π_j is defined to be $|\tau_i - \tau_j|$. However in some applications this measure may not be appropriate.

In Section 2 we describe briefly the ranking problem and the proposed rule. A proof is given for the fact that the procedure guarantees the stated probability requirements. This procedure can be easily extended to the ranking of k stochastic processes belonging to the Koopman-Darmois family. The procedure so obtained will tend to be on the conservative side in the sense of "overprotection" due to inequalities on the probability of correct selection which are used in its construction. Certain modification of the procedure for values of $k \leqq 5$ so as to reduce the amount of overprotection (and consequently, the average sample size) are obtained in Section 3. In Section 4 the procedure is applied to the problem of selecting the Poisson process with the largest parameter when $k = 3$ and the *exact* probability of correct selection is obtained using relaxation techniques.

2. Formulation of the selection problem.

2.1. *Koopman-Darmois Populations and Some of Their Relevant Properties.* A univariate population is said to belong to the Koopman-Darmois family if

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