The Annals of Mathematical Statistics
1968, Vol. 39, No. 6, 2029–2037

ON SLIPPAGE TESTS—(II) SIMILAR SLIPPAGE TESTS[1]

BY IRVING J. HALL[2], AKIO KUDÔ[3] AND NENG-CHE YEH[4]

Iowa State University and Kyushu University

1. Introduction. This is a continuation of the previous paper of Hall and Kudô [1], and all the notations and nomenclature are the same as in the previous paper. The purpose of this paper is to explore the possibility of applying the concept of similarity in hypotheses testing to slippage tests.

The authors are grateful to the referee, who suggested the strengthened version of the result in the original manuscript.

2. Similarity in exponential family of distributions. In accordance with hypotheses testing we can define a similar size α decision function.

In this section we consider some general aspects of uniformly most powerful symmetric similar size α decision functions. A decision function is said to be *similar size* if the expectation of $\varphi_0(x)$ is equal to $1 - \alpha$ whenever H_0 is true.

Let S be distributed according to the exponential family with parameter space $\Omega = \{\theta\}$ which can be divided into $a + 1$ disjoint subsets $\Omega = \Omega_0 \cup \Omega_1 \cup \cdots \cup \Omega_a$ such that $\Omega_0 \cup \Omega_i$ is covered by a family of disjoint curves originating from Ω_0, $\Omega_i = \{\theta_i(\gamma, \sigma): 0 < \gamma < \infty, \sigma \varepsilon \Omega_0\}$, $\theta_i(0, \sigma) = \sigma$ and $\theta_i(\gamma, \sigma) \varepsilon \Omega_i$ for all $\gamma \varepsilon (0, \infty)$ and σ so that the parameter can be expressed as $\theta_i(\gamma, \sigma)$ or (i, γ, σ) for $\theta \varepsilon \Omega_i$ and σ for $\theta \varepsilon \Omega_0$.

We assume that U is the minimal sufficient statistic, for Ω_0, (U, T_i) for $\Omega_0 \cup \Omega_i$, S for $\Omega_0 \cup \Omega_1 \cup \cdots \cup \Omega_a$ and that the density of S wrt μ can be expressed as

$$(1) \quad dP^S(s)/d\mu(s) = dP^S_{i,\gamma,\sigma}(s)/d\mu(s) = C(i, \gamma, \sigma) \exp [\alpha(i, \gamma, \sigma)U(s) + \beta(\gamma, \sigma)T_i(s)].$$

As before we assume there is a group $G = \{g\}$ of transformations on S isomorphic to the permutation group of $(1, 2, \cdots, a)$ itself or to its subgroup transitive on $(1, 2, \cdots, a)$ and $\mu(A) = \mu(gA)$. Let the number of elements in G be N. In addition we assume

A.1. $T_i(s) = T_{\pi_g i}(gs)$.

A.2. $U(gs_1) = U(gs_2)$ for all g if and only if $U(s_1) = U(s_2)$.

This enables us to define $G_u = \{g_u\}$, a transformation group defined on the space of U. G_u is, of course, finite and its number of elements is denoted by M.

Received 3 January 1966; revised 15 April 1968.

[1] This research was supported in part by National Science Foundation Grants GP-3918 and GM-6149. The work started while Hall and Kudô were at Iowa State University, and completed after Kudô went back to Kyushu University in cooperation with Yeh.

[2] Presently at Sandia Corporation, Albuquerque, New Mexico.

[3] Presently at Kyushu University, Fukuoka, Japan.

[4] Presently at Kyushu University and on leave from Tamkang College of Arts and Sciences, Taipei, Taiwan, China.