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ON THE ADMISSIBILITY AT ∞, WITHIN THE CLASS OF RANDOMIZED DESIGNS, OF BALANCED DESIGNS[1]

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1. Introduction. The power properties of randomized designs locally about the hypothesis set have been considered by Kiefer [4] and Farrell [1]. Our purpose here is to consider how one should choose a randomized design in order to maximize power far out. This problem is considered when the underlying model is a one way classification and when the underlying model is a two way classification without interaction. Material on the two way classification is to be thought of as being an analysis of a "complex" design problem of interest.

In this section we explain the terminology used. We then state some theorems at the end of this section. The proofs of the theorems require a lemma about convex mixtures of design vectors, proven in Section 2, a lemma about the convexity of the power function of an F-test, proven in Section 3, and estimates on the magnitude of the power of scale invariant tests, obtained in Section 4. Proofs of the theorems are given in subsections of Section 6.

We begin by describing the problem of a two way classification. We assume there are RS populations such that population (r, s) is characterized by a mean value $\varphi_{0r} + \varphi_{1s}$ and let $\varphi_0^T = (\varphi_{01}, \cdots, \varphi_{0R})$, $\varphi_1^T = (\varphi_{11}, \cdots, \varphi_{1S})$, and $\varphi^T = (\varphi_0^T, \varphi_1^T)$. A $H \times 1$ vector X is observed such that $EX = C\varphi$, the matrix $C = (A, B)$ is $H \times (R + S)$ and A is $H \times R$, B is $H \times S$. Each entry in A and B is either 0 or 1, and a given row of A and of B have exactly one non-zero entry. The matrices A and B in effect specify the design and the matrix A^TB has entries n_{rs}, the number of observations on the mean value $\varphi_{0r} + \varphi_{1s}$.

As will appear in the sequel the matrix

$$D = A^TA - (A^TB)(B^TB)^+(B^TA) \tag{1.1}$$

plays an important role in the analysis of variance theory. Corresponding to a square matrix M we let M^+ be its unique generalized inverse in the sense of Penrose [5]. Since the properties of generalized inverses are important to the calculations below we list them here for later reference.

$$\text{(1.2a)} \qquad MM^+M = M; \qquad M^+MM^+ = M^+; \qquad (M^+M)^T = M^+M; \qquad (MM^+)^T = MM^+.$$

The properties (1.2a) are known to uniquely determine the generalized inverse

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