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CONTAGION IN STOCHASTIC MODELS FOR EPIDEMICS[1]

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1. Introduction. The present paper originated in an effort to build appropriate stochastic models for the spread of epidemics and in an attempt to study their properties. The main results consist in the description of the properties of a test for contagion in an homogeneous population subject to a non-Markovian type of epidemic.

Owing to mathematical intractability, the models actually studied in the literature are always oversimplified in many respects. The present work uses also an oversimplified model. However, we do hope that the method developed in this restricted situation will be found susceptible of applications in more realistic cases.

The model considered here assumes that one observes a finite population consisting of susceptibles, infectives and immunes. A susceptible individual may be changed to infective either through general causes independent of the status of the disease in the population or through infection transmitted by the infectives of the population. It is assumed that during the time of observation the same individual will not recover enough to become infected twice.

The main difference between the model studied here and models used by other investigators is that the infectiousness of a diseased individual depends not only on the length of time he has been sick but also on the time at which he was infected with the disease. Thus to describe the state of the population at time τ it is necessary to give not only the number of infectives but also the times at which they became infected.

A precise description of the actual model studied in detail is given in Section 2. For this particular model we construct an asymptotically optimal test for contagion and investigate its properties, including asymptotic evaluations of its power. Two different sampling procedures are considered. In Section 3 the sampling method consists of observing the population until a predetermined number of cases have been observed. For this sampling procedure one obtains an asymptotic distribution for the logarithm of the likelihood ratio. This is done both under the assumption that no contagion is actually present and under the assumption that the contagion within the population is detectable but not overwhelmingly obvious.

In Section 4 similar results are obtained but the sampling procedure is different. It is assumed there that the length of time of observation is predetermined.

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