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INVARIANT PROPER BAYES TESTS FOR EXPONENTIAL FAMILIES[1]

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1. Introduction. Throughout this paper the phrase "invariant Bayes test" used without further qualification will mean a test which is (proper) Bayes among all tests and which is also invariant.

In a variety of problems arising in normal multivariate analysis Kiefer and Schwartz (1965) (hereafter K-S (1965)) have constructed fully invariant proper Bayes tests. In K-S (1965) each problem and each test for a given problem are treated somewhat separately. There is little indication of a general method of constructing *a priori* distributions which yield invariant tests or of the requirements on the problem in order that the method be successful, or of the class of tests which can be constructed in this way.

The present paper is focused on testing problems concerning the parameter of an exponential family of probability densities when the problem remains invariant under a locally compact group. *A priori* distributions are constructed in such a way that the role of the transformation groups leaving the problem invariant is clarified. Verification that the resulting Bayes tests are fully invariant does not depend on an explicit computation of the tests.

The basic idea of this paper arose from the realization that the methods used in K-S (1965) are intimately related to Stein's method of obtaining the probability density of the maximal invariant under a group, G, as an integral over G with respect to Haar measure. Although Stein's representation of the probability density of the maximal invariant motivates the construction of the *a priori* distributions in Theorem 1 of Section 3, neither the construction itself nor the invariance of the resulting Bayes tests depend upon the validity of the representation.

However, when Stein's representation is valid, the Bayes tests obtained in this paper have an interesting interpretation, and this interpretation permits the characterization of a wide class of invariant Bayes tests (Theorem 2).

In Section 2, requisite notation and definitions are given, along with background material on Stein's representation. Section 3 gives the main general results on invariant Bayes tests. One specific example, the MANOVA problem, has been worked out in Schwartz (1966) using explicit computations rather than the general results based on Stein's representation, and that paper also contains a sketch of the general results. (Without the example and background provided by Schwartz (1966) the formulation of Section 3 below will probably seem unmotivated and difficult to follow!) Section 5 is devoted to a second example:

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