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THE CENTRAL LIMIT PROBLEM FOR GENERALIZED RANDOM FIELDS

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1. Introduction. For probability measures in the real line two main results concerning sums of independent random variables are the Lèvy-Khintchine representation of an infinitely divisible probability measure, and the criteria for weak convergence of probability measures. During the last two decades or so these theorems have been extended to a variety of topological Abelian groups, including vector spaces.

We mention in particular the work of Parthasarathy, Rao, and Varadhan [5] in locally compact Abelian groups, and the earlier extension by Takano [6] of the classical theory to finite-dimensional space.

On the whole, the theory for non-locally compact Abelian groups seems incomplete, due chiefly to the lack of an adequately well-behaved Fourier transform. It is therefore natural to seek to extend the theory to non-locally-compact Abelian groups in which it can be reduced to the locally compact theory. For example, one might apply the locally compact theory to the projections of one's probability measures into the locally compact quotient groups. Among the topological vector spaces, the natural domain of this technique is of course the class of duals $\mathfrak{X}$ of nuclear spaces (and duals of strict inductive limits of nuclear spaces).

In this paper we introduce the notions of bounded variances for a double sequence of measures in $\mathfrak{X}$, weak convergence of measures, and convolution. These notions coincide with the usual definitions in the special case of finite-dimensional spaces, and seem natural in terms of applications. Taking the aforementioned approach yields the following results, in terms of our generalized notions. The class of weak limits of sums of random variables with bounded variances coincides with the class of infinitely divisible measures having covariances. If μ is any infinitely divisible measure, then the log of its Fourier transform has value at each φ in the original strict inductive limit space $\mathfrak{X}'$ given by

$$A(\varphi) - \tfrac{1}{2}\|\varphi\|^2 + \int [e^{ix(\varphi)} - 1 - ix(\varphi)]\, d\nu(x)$$

where $A \in \mathfrak{X}$, $\|\varphi\|$ is a Hilbert norm on $\mathfrak{X}'$, and ν is a σ-finite measure on $\mathfrak{X}$ which integrates the function $e^{ix(\varphi)} - 1 - ix(\varphi)$ and has finite mass outside each neighborhood of the null element.

2. The Lèvy continuity theorem. Henceforth $\mathfrak{X}$ will denote the dual space of a strict inductive limit $\mathfrak{X}'$ of nuclear spaces $\mathfrak{N}_1, \mathfrak{N}_2, \cdots$; $\mathfrak{X}$ is given the strong topology. By a *measure* in $\mathfrak{X}$ we mean a σ-finite non-negative regular Borel measure ν with values in $[0, \infty]$, such that for some closed set $\mathfrak{N}$ with $\nu(\mathfrak{N}) = 0$,

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