The Annals of Mathematical Statistics
1969, Vol. 40, No. 1, 24–39

ON ROBUST LINEAR ESTIMATORS

BY JOSEPH L. GASTWIRTH[1] AND HERMAN RUBIN[2]

The Johns Hopkins University and Purdue University

1. Introduction. The problem of finding robust estimators for the location parameter of symmetric unimodal distributions has been the subject of much recent research (e.g. [2], [3], [6], [8], [9], [13], [15]). This paper is concerned with finding robust estimators which are linear functions of the ordered observations. Thus, the robust estimators proposed by Hodges and Lehmann [8] and Huber [9] are not considered in detail. The spirit of the present work is similar to a previous paper of one of the authors [7] and is also related to the fundamental work of Tukey [13], [14].

We assume that the density function of the population sampled is a member of a class $\mathfrak{F}$ of densities. For every member f_γ (γ runs through an index set Γ), there is an asymptotically efficient estimator S_γ which is a linear combination of the ordered observations [1], [5], [10]. The asymptotic efficiency of any estimator D for samples from the density f_γ is the reciprocal of the ratio of the asymptotic variance of D to the asymptotic variance of S_γ. Throughout the paper the asymptotic variance of an estimator will mean the variance of its asymptotic normal distribution.

An estimator will be called a maximin efficient estimator within a class C of estimators for a family of densities if it maximizes the minimum asymptotic efficiency over the family $\mathfrak{F}$. In Section 2, we demonstrate that within a large class C of linear estimators, there is a unique maximin efficient linear estimator for general families of densities. Under somewhat more restrictive conditions on the family $\mathfrak{F}$ of possible densities we show that within the class C of linear estimators the Bayes estimators are the minimal complete class. These results are asymptotic generalizations of the work of Birnbaum and Laska [3].

In Section 3 we discuss, in detail, the special case when $\mathfrak{F}$ contains the logistic and double-exponential distributions. The maximin efficient linear estimator (m.e.l.e.) is found and is compared to the best convex combination of the individual optimum linear estimator and also to a Hodges-Lehmann type estimator based on the corresponding maximin rank test [7].

In general, the m.e.l.e. for specific families of densities is quite difficult to compute. It seems appropriate, therefore, to look for a maximin efficient estimator in smaller classes of linear estimators which are easy to use. Two such

Received 1 June 1967; revised 15 May 1968.

[1] Research supported by the National Science Foundation Research Grant No. GP 6225 awarded to the Department of Statistics, The Johns Hopkins University. This paper in whole or in part may be reproduced for any purpose of the University State Government.

[2] Research supported by the Office of Naval Research under contract NONR 2587 (02) awarded to the Department of Statistics, Michigan State University. This paper in whole or in part may be reproduced for any purpose of the United States Government.