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# A NOTE ON BALANCED INCOMPLETE BLOCK DESIGNS[1]

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The notation and terminology of [2] will be used in this paper.

Many ingenious methods for the construction of balanced incomplete block designs have been devised but little attention has been given to conditions under which the blocks of such a design are all distinct. The reason for this is probably that repetition of a block does not affect the usefulness of an experimental design. Repetition of some blocks in a design while others are not repeated is however an interesting combinatorial property and in some possible applications like coding might even affect the practical use of the design. The special case with parameters $v, b, r, k, \lambda = 2x + 2, 4x + 2, 2x + 1, x + 1, x$ was investigated by E. T. Parker [3] and by Esther Seiden [4]. Parker also gave a non-trivial example of a balanced incomplete block design with repetitions of some of the blocks.

We shall prove the following theorem.

THEOREM. *Let $D$ be a balanced incomplete block design with parameters $v, b, r, k, \lambda$. If $s$ blocks of $D$ are identical and if $r > \lambda$ then*

$$r/k = b/v \geqq s. \tag{1}$$

The theorem implies the results of E. Parker and of Esther Seiden. Its proof is an adaptation of R. A. Fisher's proof for the special case $s = 1$.

PROOF OF THE THEOREM. Assume that the first $s$ blocks $B_1, B_2, \cdots, B_s$ are identical. Let $a_i$ be the number of elements common to the $i$th block $i > s$ and to $B_1$. Then

$$\sum a_i = (r - s)k,$$
$$\sum a_i(a_i - 1) = \sum a_i^2 - \sum a_i = (\lambda - s)k(k - 1).$$

Hence

$$\sum a_i^2 = k[(\lambda - s)k + (r - \lambda)] \geqq (r - s)^2k^2/(b - s),$$
$$(\lambda - s)k + (r - \lambda) \geqq (r - s)^2k/(b - s),$$
$$[(b - r) + (r - s)][(r - s)k - (r - \lambda)k + (r - \lambda)] \geqq (r - s)^2k,$$
$$(r - s)[(b - r)k - (r - \lambda)k + (r - \lambda)] \geqq (b - r)(r - \lambda)(k - 1).$$

Replace $bk$ by $rv$ and $r - \lambda$ by $rk - \lambda v$ then

$$(r - s)((r - \lambda)v - (r - \lambda)k) \geqq (b - r)(r - \lambda)(k - 1),$$
$$r - s \geqq (b - r)(v - k)^{-1}(k - 1) = rk^{-1}(k - 1).$$

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