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AN EXTENSION OF A THEOREM OF CHOW AND ROBBINS ON SEQUENTIAL CONFIDENCE INTERVALS FOR THE MEAN[1]

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1. Introduction. Let $X_1, X_2, \cdots$ be a sequence of independent observations from a population with mean μ and finite nonzero variance σ^2. We wish to estimate the unknown μ by confidence intervals of prescribed "accuracy" and prescribed probability of coverage α. Let

$$(1) \qquad \bar{X}_n = n^{-1} \textstyle\sum_{k=1}^n X_k, \qquad (n = 1, 2, \cdots).$$

We speak of "absolute accuracy" when estimating μ by

$$(2) \qquad I_n = (\mu : |\bar{X}_n - \mu| \leqq d), \qquad (d > 0),$$

and, if $\mu \neq 0$, we speak of "proportional accuracy" when estimating μ by

$$(3) \qquad J_n = (\mu : |\bar{X}_n - \mu| \leqq p\,|\mu|), \qquad (0 < p < 1).$$

Denote by ρ the coefficient of variation $\sigma/|\mu|$ and define

$$(4) \qquad n(d) = \min_{n \geqq 1} (n : \sigma^2 \leqq n(d/a)^2),$$

$$(5) \qquad m(p) = \min_{n \geqq 1} (n : \rho^2 \leqq n(p/a)^2)$$

where a is the $\frac{1}{2}(1 - \alpha)$th fractile of the standard normal distribution. Then (4) and (5) increase without bound as the arguments tend to zero. Hence for small arguments we can achieve (at least approximately) the required probability of coverage α by taking the "sample size" n no smaller than $n(d)$ (for absolute accuracy) or $m(p)$ (for proportional accuracy).

If, however, σ^2 (the "appropriate parameter" for absolute accuracy) is unknown, or if ρ^2 (the "appropriate parameter" for proportional accuracy) is unknown then (4) or (5) are not available. On the other hand if we let

$$(6) \qquad V_n^2 = n^{-1}(1 + \textstyle\sum_{k=1}^n (X_k - \bar{X}_n)^2), \qquad (n = 1, 2, \cdots),$$

then the stopping rules

$$(7) \qquad N = \min_{n \geqq 1} (n : V_n^2 \leqq n(d/a)^2)$$

and

$$(8) \qquad M = \min_{n \geqq 1} (n : (V_n/\bar{X}_n)^2 \leqq n(p/a)^2)$$

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