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APPROXIMATION TO BAYES RISK IN SEQUENCES OF NON-FINITE GAMES[1]

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1. Introduction. This paper is concerned with the product of a game. The main result is the demonstration of a sequence strategy for player II which results in average risk across the first n plays approaching uniformly the Bayes envelope evaluated at the empirical distribution of player I's first n moves.

We consider a two-person game where player I chooses an $\epsilon \varepsilon M$ and player II chooses $\delta \varepsilon N$ with loss $L(\epsilon, \delta) \geqq 0$. A compact notation is provided by defining the set of loss functions $N^* = \{L(\cdot, \delta) \mid \delta \varepsilon N\}$. We let σ denote a generic element of N^* and $\epsilon\sigma$ denote σ evaluated at ϵ. This extends to operator notation $w\sigma = \int \epsilon\sigma w(d\epsilon)$ for measures w on M. The Bayes envelope is defined by

$$R(p) = \inf \{p\sigma \mid \sigma \varepsilon N^*\}$$

where p is a probability measure on M (a mixed strategy for player I).

We suppose that this game occurs repeatedly, ϵ_i represents player I's move at the ith stage, and G_{i-1}, the empirical distribution of $\epsilon_1, \cdots, \epsilon_{i-1}$, is known to player II before he makes his move at the ith stage, $i \geqq 2$. In this paper we let G_0 denote the zero measure and demonstrate sequence strategies $\boldsymbol{\delta} = (\sigma_1, \sigma_2, \cdots)$ for player II, where σ_i depends upon G_{i-1} and some artificial randomization, such that $n^{-1} \sum_1^n E(\epsilon_i\sigma_i) - R(G_n) \rightarrow 0$ as $n \rightarrow \infty$ uniformly in $\boldsymbol{\epsilon}$.

The notion of using the Bayes envelope as an asymptotic standard in a set of statistical decision problems, with statistical information on G_n replacing knowledge of G_{i-1} in the sequence case, is due to Robbins [11]. Since Robbins' original investigation, procedures which achieve the Bayes envelope asymptotically have been demonstrated and rates of convergence investigated for sequences and sets of a variety of statistical decision problem components, [3], [4], [5], [6], [8], [9], [10], [12], [13], [14], [15], [16], [17] and [18]. However, this paper treats the problem at a game theoretic level and is more closely related to the studies [1], [2], and [7]. This paper depends heavily upon the notation and ideas of Hannan [7], but an effort is made to keep the presentation self-contained.

2. The main result. We impose the following condition on the component game:

(A1) N^* is sequentially compact under pointwise convergence.

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