

ON THE A PRIORI DISTRIBUTION OF THE COVARIANCE MATRIX¹

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The purpose of this note is to give a fiducial argument in support of the well known non-informative *a priori* distribution of a covariance matrix. Although the fiducial argument was introduced and promoted energetically by R. A. Fisher as an inference method which presumably would lead to necessary (and therefore unique) *a posteriori* distributions [4], it was discovered soon that different fiducial arguments could lead to entirely different *a posteriori* distributions [2], and, in the particular case of fiducial inferences about a covariance matrix, interesting problems arise which require further research [3].

Let X be a random $p \times p$ matrix, which has the multivariate normal distribution with mean value equal to 0 (the null matrix) and covariance matrix Σ . That is, the density of X is proportional to

$$(1) \quad |\Sigma|^{-p/2} \exp -\frac{1}{2} \text{tr} \Sigma^{-1} X X'.$$

Consider the change of variable

$$(2) \quad Y = \Sigma^{-1/2} X.$$

As is well known, the random matrix Y has the multivariate normal distribution with mean value equal to 0 and covariance matrix I (the identity matrix). Obviously, the transpose Y' has the same distribution, and

$$(3) \quad W = Y'Y = X' \Sigma^{-1} X$$

has the Wishart distribution with a mean value equal to pI , covariance matrix I , and p degrees of freedom, the density of which is proportional to

$$(4) \quad |W|^{-1/2} \exp -\frac{1}{2} \text{tr} W.$$

Suppose now that X is an observed matrix and that Σ is unknown. Since the distribution of W does not depend on Σ , it is natural to assume that the *a posteriori* distribution of W is the same Wishart distribution. *A posteriori*, X and X' are constant matrices, and (3) is a transformation between the symmetric matrices W and Σ^{-1} , the Jacobian of which is, according to Theorem 3.7 of [1],

$$(5) \quad dW/d\Sigma^{-1} = |X|^{p+1}.$$

Hence the *a posteriori* density of Σ^{-1} is, up to a constant factor,

$$(6) \quad |\Sigma|^{1/2} \exp -\frac{1}{2} \text{tr} \Sigma^{-1} X X'.$$

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