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A NOTE ON SEQUENCES OF CONTINUOUS PARAMETER MARKOV CHAINS

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We will consider continuous parameter Markov chains which for convenience, all have the same state space $S = \{1, 2, 3, \cdots\}$. We will assume that the transition probabilities

$$p_{ij}(t) = P\{X(t) = j \mid X(0) = i\}$$

satisfy the usual conditions

$$p_{ij}(t) \geqq 0, \tag{1}$$

$$\sum_{j=1}^{\infty} p_{ij}(t) \leqq 1, \tag{2}$$

$$\sum_{k=1}^{\infty} p_{ik}(s) p_{kj}(t) = p_{ij}(s + t), \tag{3}$$

and

$$\lim_{t \to 0} p_{ij}(t) = \delta_{ij}. \tag{4}$$

These conditions imply the continuous differentiability of $p_{ij}(t)$ (see Chung [1]), and we define

$$q_{ij} = p'_{ij}(0). \tag{5}$$

The q_{ij} satisfy

$$0 \leqq q_{ij} < \infty \qquad \text{for} \quad i \neq j \tag{6}$$

and

$$\sum_{j=1}^{\infty} q_{ij} \leqq 0. \tag{7}$$

In addition we will assume

$$q_i \equiv -q_{ii} < \infty. \tag{8}$$

For a given matrix $Q = ((q_{ij}))$, whose elements satisfy (6)–(8), there exists at least one matrix of transition probabilities $P(t) = ((p_{ij}(t)))$ satisfying (1)–(5). Any such matrix is called a Q-transition matrix, and a Markov chain with these transition probabilities is called a Q-process.

We are interested in the behavior of a sequence of transition matrices $P_n(t) = ((p_{ij}^n(t)))$ with corresponding matrices $Q^n = ((q_{ij}^n))$ satisfying (1)–(8) under the assumptions that

$$\lim_{n \to \infty} q_{ij}^n = q_{ij} \qquad \text{exists for all } i \text{ and } j, \tag{9}$$

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