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ON LIMITING DISTRIBUTIONS FOR SUMS OF A RANDOM NUMBER OF INDEPENDENT RANDOM VECTORS[1]

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1. Introduction. Consider a sequence of $p \times 1$ random (column) vectors $\{y_n\}$, $n = 1, 2, \cdots$. Suppose that there exists a sequence $\{B_n\}$ of real $p \times p$ nonsingular matrices and a proper p-variate distribution function $F(y)$ such that

$$\lim_{n\to\infty} \mathfrak{L}(B_n^{-1} y_n) = \mathfrak{L}(y^*), \tag{1.1}$$

where y^* is a $p \times 1$ random vector having the distribution function $F(y)$. (The notation $\mathfrak{L}(y^*)$ denotes the law or distribution of y^*. $\text{Lim}_{n\to\infty} \mathfrak{L}(Z_n) = \mathfrak{L}(Z)$ means that Z_n converges in law (converges weakly) to Z. The notation $\mathfrak{L}(\mathfrak{N}(0, \sigma^2 I))$ used later is short for the law of a multivariate normal random variable with mean vector 0 and covariance matrix $\sigma^2 I$.) Suppose further that we have an infinite sequence $\{\nu_n\}$, $n = 1, 2, \cdots$, of positive integer-valued random variables, and a sequence $\{k_n\}$ of positive integers such that

$$\lim_{n\to\infty} k_n = \infty, \qquad \text{plim}_{n\to\infty} k_n^{-1} \nu_n = 1. \tag{1.2}$$

We are interested in conditions under which

$$\lim_{n\to\infty} \mathfrak{L}(B_{k_n}^{-1} y_{\nu_n}) = \mathfrak{L}(y^*). \tag{1.3}$$

In the scalar case ($p = 1$), Anscombe [2] found a sufficient condition for (1.3) to hold. One extension of that theorem (Theorem 1 of [2]) to the vector case ($p > 1$) is the following.

THEOREM 1.1. *If the sequences* $\{y_n\}$, $\{B_n\}$, $\{\nu_n\}$, *and* $\{k_n\}$ *satisfy* (1.1) *and* (1.2), *then for* (1.3) *to hold, it is sufficient that for given* $\epsilon > 0$, $\eta > 0$, *there exists a positive integer* n_0 *and a positive number* c *such that for all* $n \geqq n_0$,

$$P\{\max_{n' : |n-n'| < cn} \|B_n^{-1}(y_n - y_{n'})\|_2 < \epsilon\} > 1 - \eta. \tag{1.4}$$

Here, for a $p \times 1$ *vector* $Z = (Z_1, Z_2, \cdots, Z_p)'$, *the notation* $\|Z\|_2$ *represents the* L_2 *norm of* Z, i.e., $\|Z\|_2 = (Z'Z)^{\frac{1}{2}}$. *The notation* $\|Z\|_\infty$ *is used to represent the* L_∞ *norm of* Z, i.e., $\|Z\|_\infty = \max_{1 \leqq j \leqq p} |Z_j|$.

NOTE. We note that nothing is supposed concerning the dependence of ν_n on the random vectors y_k.

Theorem 1.1 is proven in Section 2. The proof closely resembles that given by Anscombe [2] in the scalar case, and consequently is only briefly sketched.

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