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A NOTE ON THE TEST FOR THE LOCATION PARAMETER OF AN EXPONENTIAL DISTRIBUTION

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Suppose that X is distributed according to an exponential distribution with density

$$f(x) = \theta^{-1} \exp [-\theta^{-1}(x - \gamma)] \quad \text{if} \quad x > \gamma, \qquad (1)$$
$$= 0 \quad \text{otherwise}$$

where both θ and γ are unknown parameters.

Let Y be another positive random variable independent of X and distributed according to a continuous distribution with scale parameter θ, and density $g(y/\theta)/\theta$, where g is a known function.

Consider the hypothesis $\gamma = \gamma_0$, and let the procedure be such that the hypothesis is rejected if and only if

$$(X - \gamma_0)/Y \leqq a \quad \text{or} \quad (X - \gamma_0)/Y \geqq b \qquad (2)$$

where $0 \leqq a < b \leqq \infty$. Since $(X - \gamma_0)/Y$ is independent of θ under the hypothesis, a and b can be determined so that

$$\Pr \{a < (X - \gamma_0)/Y < b \mid \gamma_0\} = 1 - \alpha \quad \text{for all} \quad \theta.$$

Then the following theorem holds true.

Theorem 1. *For the alternative $\gamma < \gamma_0$, the power of the test (2) above is given by*

$$P(\gamma) = 1 - (1 - \alpha) \exp [-\theta^{-1}(\gamma_0 - \gamma)] \qquad (3)$$

i.e. it is independent of the distribution of Y, and also of a or b.

Proof.

$$P(\gamma) = 1 - P_\gamma\{\gamma_0 + aY < X < \gamma_0 + bY\}$$
$$= 1 - P_\gamma\{\gamma_0 - \gamma + aY < X - \gamma < \gamma_0 - \gamma + bY\}$$
$$= 1 - \int_0^\infty [\int_{\gamma_0-\gamma+ay}^{\gamma_0-\gamma+by} \theta^{-1} \exp (-\theta^{-1}u) \, du]\theta^{-1} g(\theta^{-1}y) \, dy$$
$$= 1 - \int_0^\infty \theta^{-1} | \exp [-\theta^{-1}(\gamma_0 - \gamma + ay)] - \exp [-\theta^{-1}(\gamma_0 - \gamma + by)| \; g(\theta^{-1}y) \, dy$$
$$= 1 - \exp [-\theta^{-1}(\gamma_0 - \gamma)] \int_0^\infty \theta^{-1} (\exp (-\theta^{-1}ay) - \exp (-\theta^{-1}by)) g(\theta^{-1}y) \, dy$$
$$= 1 - \exp [-\theta^{-1}(\gamma_0 - \gamma)] \int_0^\infty (e^{-au} - e^{-bu}) g(u) \, du$$

Since $P(\gamma_0) = \alpha$, the theorem is proved.

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