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NOTES

RESULTS FROM THE RELATION BETWEEN TWO STATISTICS OF THE KOLMOGOROV-SMIRNOV TYPE

BY M. A. STEPHENS

McGill University and University of Nottingham

1. Introduction. In this paper we demonstrate the relation existing between the distributions of two statistics of the Kolmogorov-Smirnov type. They have been named K_n (Brunk, 1962) and V_n (Kuiper, 1960; Stephens, 1965). From the relation, given as a theorem in Section 3, new results are found for each statistic by using what is known about the other. Tables of percentage points are given for K_n, and it is shown how to adapt an existing table for K_n to give a table of probabilities for V_n.

2. The statistic V_n. This is a statistic of the Kolmogorov-Smirnov type, suitable for tests of goodness-of-fit. Suppose a random sample of size n is given, and let the values, in ascending order, be $x_1, x_2, \cdots, x_n$; let the sample or empirical distribution function be $F_n(x)$. It is required to test the null hypothesis H_0, that the sample comes from a continuous distribution $F(x)$; well-known test statistics are

$$D_n^+ = \sup_{-\infty<x<\infty} \{F_n(x) - F(x)\}$$

$$D_n^- = \sup_{-\infty<x<\infty} \{F(x) - F_n(x)\}$$

$$D_n = \max \{D_n^+, D_n^-\}.$$

V_n is given by $D_n^+ + D_n^-$. It was suggested by Kuiper (1960) for use with observations on a circle; the value of V_n does not depend on the choice of origin for x. This is a necessary property of a goodness-of-fit statistic for the circle, since otherwise the same data could, by a change of origin, yield different values of the test statistic. V_n may, of course, be used also for observations on a line. The asymptotic distribution of V_n was given by Kuiper, and the small-sample distribution, in the tails, by Stephens (1965); in the latter paper there are tables of upper and lower percentage points of V_n.

3. The statistic K_n. Suppose, following Brunk's (1962) notation, that $U_i = F(x_i)$, $i = 1, 2, \cdots, n$; put $U_0 = 0$, $U_{n+1} = 1$. Define statistics (the *C-class*):

$$C_n^+ = \max_{0\leq i\leq n+1} (i/(n+1) - U_i),$$

$$C_n^- = \max_{0\leq i\leq n+1} (U_i - i/(n+1)),$$

$$C_n = \max \{C_n^+, C_n^-\} \quad \text{and} \quad K_n = C_n^+ + C_n^-.$$

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