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ASYMPTOTIC LINEARITY OF A RANK STATISTIC IN REGRESSION PARAMETER

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0. Introduction. Let $(X_1, X_2, \cdots, X_N)$ be an independent random sample from a distribution with finite Fisher's information and let us consider the statistic

$$S_{\Delta N} = \sum_{i=1}^N c_i a_N(R_{Ni}^\Delta)$$

where $R_{N1}^\Delta, R_{N2}^\Delta, \cdots, R_{NN}^\Delta$ is the vector of ranks for random variables $X_1 + \Delta d_1$, $X_2 + \Delta d_2, \cdots, X_N + \Delta d_N$; Δ, c_i and d_i, $1 \leqq i \leqq N$ are real constants. Then $\{S_{\Delta N}; -\infty < \Delta < \infty\}$ forms a random process. We show at first that under some assumptions the realizations of this process are monotone step-functions of Δ and that these realizations are asymptotically linear in Δ in the sense of the formula (3.1) of Theorem 3.1. The asymptotic linearity of $S_{\Delta N}$ may be proved also in the case of K–variate regression, when instead of R_{Ni}^Δ's there will occur the ranks of the values $X_1 + \Delta_1 d_{11} + \Delta_2 d_{21} + \cdots + \Delta_K d_{K1}, \cdots, X_N + \Delta_1 d_{1N} + \cdots + \Delta_K d_{KN}$; the statistic $S_{\Delta N}$ is then an asymptotically linear function of the parameters $\Delta_1, \Delta_2, \cdots, \Delta_K$.

Some possibilities of application are mentioned.

1. Notation and basic assumptions. We shall consider for any positive integer N:

(a) an independent random sample $(X_{N1}, X_{N2}, \cdots, X_{NN})$ from a distribution whose distribution function F has finite Fisher's information, i.e.

$$\int_{-\infty}^{\infty} [f'(x)/f(x)]^2 f(x)\, dx < \infty,$$

where f is the density of the distribution;

(b) a real vector $(c_{N1}, c_{N2}, \cdots, c_{NN})$ (so called regression constants) such that

$$\sum_{i=1}^N (c_{Ni} - \bar{c}_N)^2 > 0. \tag{1.1}$$

$$\lim_{N\to\infty} \max_{1\leqq i\leqq N} (c_{Ni} - \bar{c}_N)^2 \cdot \left[\sum_{j=1}^N (c_{Nj} - \bar{c}_N)^2\right]^{-1} = 0 \tag{1.2}$$

where $\bar{c}_N = (1/N) \sum_{i=1}^N c_{Ni}$.
Condition (1.2) is the so called Noether's condition.

(c) a real vector $(d_{N1}, d_{N2}, \cdots, d_{NN})$ such that

$$\sum_{i=1}^N (d_{Ni} - \bar{d}_N)^2 \leqq M \quad \text{for} \quad N = 1, 2, \cdots \tag{1.3}$$

where $M > 0$ is a constant, $\bar{d}_N = (1/N) \sum_{i=1}^N d_{Ni}$ and

$$\max_{1\leqq i\leqq N} (d_{Ni} - \bar{d}_N)^2 \to 0 \quad \text{for} \quad N \to \infty. \tag{1.4}$$

(d) a real parameter Δ.

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