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ABSTRACTS OF PAPERS

(*Abstracts of papers to be presented at the Central Regional meeting, Dallas, Texas, April* 8–10, 1970. *Additional abstracts will appear in future issues.*)

124-2. A class of conjugate prior distributions, and optimal allocation: a multivariate extension. RONNIE L. MORGAN, Oregon State University.

An extension to a paper entitled "A class of conjugate prior distributions, and optimal allocation" contributed by the author at the 1969 region Spring Meeting of the Joint Statistical Conference at Iowa City, Iowa is given here. A univariate one-parameter family of sampling distributions of the form $f(x \mid \tau) = \exp[x\tau - \theta(\tau)]f(x)$ was considered in the earlier paper. The multivariate family defined for a p-dimensional variate $\mathbf{x} = (x_1, x_2, \cdots, x_p)$ by $f(\mathbf{x} \mid \tau) = \exp[\sum_{i=1}^{m} \varphi_i(\mathbf{x})\tau_i - \theta(\tau)]f(\mathbf{x})$ is considered here. A class of conjugate priors of the form $g(\tau) \propto \exp[\mathbf{a}'\tau - b\theta(\tau)]$ is assumed. Let $\theta_i(\tau) = \partial\theta(\tau)/\partial\tau_i$, and $\theta_{ij}(\tau) = \partial^2\theta(\tau)/\partial\tau_i\,\partial\tau_j$, then $\theta_i(\tau) = E[\varphi_i(\tilde{\mathbf{x}})]$ and $\theta_{ij}(\tau) = \mathrm{Cov}[\varphi_i(\mathbf{x}), \varphi_j(\mathbf{x})]$. The following theorems are proved, under certain regularity conditions. THEOREM 1. $E[\theta_i(\tilde{\tau})] = a_i/b$. THEOREM 2. $E[\theta_{ij}(\tilde{\tau})] = b\,\mathrm{Cov}[\theta_i(\tilde{\tau}), \theta_j(\tilde{\tau})]$. THEOREM 3. $E[\mathrm{Cov}\{\theta_i(\tilde{\tau}), \theta_j(\tilde{\tau})\}] = b\,\mathrm{Cov}[\theta_i(\tilde{\tau}), \theta_j(\tilde{\tau})]/(n+b)$. The expectation in Theorem 3 is with respect to the marginal distribution of $\mathbf{x}$ as determined by the assumed prior and Cov denotes the posterior covariance. The results of this paper are applied to several optimal allocation problems. In particular when application is made to optimal stratified sampling, the resulting solution is shown to be a generalization of the Neyman allocation formula. (Received October 14, 1969.)

124-3. Bulk queues with phase input. ASHA S. KAPADIA, Arthur D. Little, Inc.

In this paper the expected number of idle servers in the steady state has been obtained for the following three queues. (i) $P_{\{q_i\}}/P/1$, (ii) $P_{(K)}/P_{(N)}/1$, (iii) $P_{\{q_i\}}/M/k$. Here P represents the phase distribution (defined as the distribution of a random sum of exponential random variables).

$A_{\{q_i\}}/B/k$ refers to a queuing system which has interarrival distribution A, service time distribution B, number of servers k, batch arrival distribution $\{q_i\}\cdot(q_i =$ probability an arrival batch has i customers in it). $A_{(L)}/B_{(N)}/k$ represents batch arrivals of fixed size L and batch service of fixed size N. (Received October 30, 1969.)

(*Abstracts of papers to be presented at the Eastern Regional meeting, Chapel Hill, North Carolina, May* 5–7, 1970. *Additional abstracts will appear in future issues.*)

125-3. Queuing with general and special service. ASHA S. KAPADIA, Arthur D. Little, Inc.

Consider a system with k general and m special servers. Customers entering the system require either special or general service and they queue up in front of the special and general servers respectively. Whenever a general server is idle, he takes in a customer from the special queue, provided there are more than m customers in it. However, when a special server is idle, he cannot take in a general customer for service because special servers can perform special service only. The interarrival times of general and special customers have phase distribution (defined as the distribution of a random sum of exponential random variables) and the service time distribution for each customer is assumed exponential. It is interesting to observe that from the point of view of expected number of idle servers in the steady state, the system behaves as though the servers have no interaction where as in fact they sometimes do. The result is generalized to the case of