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BAYES AND FIDUCIAL EQUIVARIANT ESTIMATORS OF THE COMMON MEAN OF TWO NORMAL DISTRIBUTIONS[1]

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1. Introduction. The problem of pooling observations which are distributed according to different distribution laws, for the purpose of estimating a common parameter has been studied in many papers. The more specific problem of estimating the common mean of two normal distributions, with an emphasis on small sample estimators, was previously studied by the author [9], and more recently by Gurland and Mehta [4]. The reader can find in these articles relevant reference lists. Both Zacks, and Gurland and Mehta, studied procedures of combining the sample mean, by some weighted average estimators of the form:

$$\tilde{\mu} = \bar{X}\phi(S_2/S_1) + \bar{Y}(1-\phi(S_2/S_1)), \tag{1.1}$$

where $\bar{X}$ and $\bar{Y}$ are the sample means; S_1 and S_2 are the sample sum of squares of deviations, respectively. These authors confined their attention to the cases of equal sample sizes. We notice that for all choices of weighing functions $\phi(S_2/S_1)$ the above estimators are *unbiased* and invariant with respect to translation and change of scale. Gurland and Mehta showed in [4] that if it is known which one of the two distributions has the smaller variance, although the actual variance ratio is unknown, certain of the estimators suggested by Zacks in [9] can be improved upon uniformly. In the present study we investigate the whole problem more systematically in a decision theoretic framework. We start by characterizing the class of all estimators of the common mean, μ, which are translation invariant and scale preserving. Following Wijsmann [8] and Berk [1], we call these estimators *equivariant estimators*. The sample variance ratio S_2/S_1 is not the maximal invariant statistic for the group of translations and change of scale. Thus, the class of all estimators of the form (1.1) is only a subclass of the class of all equivariant estimators, and many of the estimators of the form (1.1) discussed in the previous studies are inadmissible even among the equivariant estimators. The general form of all equivariant estimators is:

$$\hat{\mu} = \bar{X} + (\bar{Y}-\bar{X})\psi(S_1(\bar{X}-\bar{Y})^{-2}, S_2(\bar{Y}-\bar{X})^{-2}). \tag{1.2}$$

The problem of choosing an equivariant estimator is equivalent to the problem of choosing a (properly measurable) function $\psi(\cdot,\cdot)$ of the maximal invariant $(S_1(\bar{Y}-\bar{X})^{-2}, S_2(\bar{Y}-\bar{X})^{-2})$. Adopting a quadratic loss function, which yields a risk function proportional to the mean-square-error risk function, in Section 3 we determine the class of all *Bayes equivariant* estimators. These are the equivariant estimators of the form (1.2), which minimize the prior risk functions corresponding

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