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THE EXIT DISTRIBUTION OF AN INTERVAL FOR COMPLETELY ASYMMETRIC STABLE PROCESSES[1]

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1. Main results. Let X_t be the stable process on the line R having exponent $\alpha \neq 1$ and log characteristic function

$$\log E \exp\left[i\theta(X_t - X_0)\right] = -t|\theta|^\alpha \left[1 - i \operatorname{sgn}(\theta) \tan(\tfrac{1}{2}\pi\alpha)\right] \tag{1.1}$$

We will assume that X_t is a version of the process that is a standard Markov process. Let $a < b$ and let $\tau = \inf\{t > 0 : X_t \notin (a, b)\}$ be the first exit time from the open interval (a, b). Our primary purpose in this note is to explicitly compute the distribution of X_τ as well as the related Green's function of $R - (a, b)$.

The results we obtain here are new for $\alpha > 1$. For $\alpha < 1$ the distribution of X_τ was first computed by Dynkin [2] and by a different method by Ikeda and Watanabe [3]. For the sake of completeness we will show how the potential theoretic methods used here also yield a very easy derivation for the case $\alpha < 1$. The results we obtain here should be compared with those of Blumenthal, Getoor, and Ray [1] for the isotropic case.

THEOREM 1. *Let $\mu_x(dy) = P_x(X_\tau \in dy)$. If $\alpha < 1$, then μ_x is the unit mass at x if $x \notin [a, b)$, while for $x \in [a, b)$*

$$\begin{aligned} \mu_x(dy) &= (\sin \pi\alpha/\pi)[(b-x)/(y-b)]^\alpha (y-x)^{-1}, && y > b \\ &= 0, \quad \textit{elsewhere}. \end{aligned} \tag{1.2}$$

On the other hand if $\alpha > 1$ and $x \in (a, b)$

$$\mu_x(\{a\}) = [(b-x)/(b-a)]^{\alpha-1} \tag{1.3}$$

$$\begin{aligned} \mu_x(dy) &= \pi^{-1} \sin[(\alpha-1)\pi][(b-x)/(y-b)]^{\alpha-1} \cdot \\ &\quad (y-x)^{-1}[(x-a)/(y-a)], && y > b \\ &= 0, && y \notin \{a\} \cup [b, \infty). \end{aligned} \tag{1.4}$$

For $x \notin (a, b)$, $\mu_x(dy)$ is the unit mass at x.

Let B be a Borel subset of (a, b). The Green's function of $R - (a, b)$ is the function $G(x, y)$ such that

$$E_x \int_0^\tau 1_B(X_t)\, dt = \int_B G(x, y)\, dy,$$

where 1_B is the indicator function of B.

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