The Annals of Mathematical Statistics
1970, Vol. 41, No. 1, 30–38

ON THE ABSOLUTE CONTINUITY OF MEASURES[1]

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1. Introduction. The problem of determining absolute continuity of measures on function spaces has been investigated for some time. Much effort has been devoted to the problem of obtaining criteria for the absolute continuity of Gaussian measures for example [13, 16, 19, 21, 23]. A well-known dichotomy for absolute continuity of Gaussian measures exists [5, 7], and some useful complete results exist for certain Gaussian measures. More recently there has been interest in obtaining conditions for absolute continuity of measures which correspond to solutions of stochastic differential equations.

We shall consider the problem of absolute continuity for processes with a continuity property on certain sub-σ-fields of the processes and indicate some simple structure on the Radon-Nikodym derivative by using the Doob-Meyer results for decomposition of supermartingales [15].

Our results will simplify and clarify some results for absolute continuity for measures corresponding to solutions of stochastic differential equations and for Gaussian measures equivalent to Wiener measure. For Gaussian measures equivalent to Wiener measure we shall relate the Gaussian process to Brownian motion via a stochastic differential equation. In this manner we obtain a "nice" transformation of Brownian motion to the other Gaussian process. The Radon-Nikodym derivative is also conveniently expressed.

2. Some general comments. When determining absolute continuity of measures we typically start from either discrete time or from some continuous time results where we know we have absolute continuity, and then try to take an appropriate limit. We have then either a martingale sequence or a martingale net, and in both cases we have necessary and sufficient conditions for absolute continuity in terms of uniform integrability (Doob [2], Helms [8]); in other words, conditions that the limit be a martingale. However, in the case of a martingale net we cannot immediately assert pointwise convergence (Dieudonné [1], Helms [8]). The importance of uniform integrability is also seen in the supermartingale work of Meyer [15].

Before discussing some results on absolute continuity, a few preliminaries will be useful. We first give a characterization of uniform integrability due to LaVallee Poussin (cf. Meyer [15]).

THEOREM 1. *Let H be a subset of L^1 $(\Omega, \mathscr{F}, P)$. The following properties are equivalent.*

(1) *H is uniformly integrable.*

Received May 1, 1969.

[1] This research was supported by United States Air Force Grant AF-AFOSR 814-66