The Annals of Mathematical Statistics
1970, Vol. 41, No. 2, 713–717

A MULTIVARIATE DEFINITION FOR INCREASING HAZARD RATE DISTRIBUTION FUNCTIONS

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1. Introduction. Increasing hazard rate (IHR) distribution functions of one variable have been discussed in the literature for many years and many of their properties have been obtained, for example, see [2]. In this paper, a definition which extends the notion of IHR to multivariate distributions is given and it is shown to satisfy certain desirable multivariate properties.

2. Multivariate IHR Distributions. Consider the random vector $(X_1, X_2, \cdots, X_n)$ with distribution function $F(x_1, x_2, \cdots, x_n) = P[X_1 \leqq x_1, X_2 \leqq x_2, \cdots, X_n \leqq x_n]$. We say that the set of random variables $X_1, X_2, \cdots X_n$ is *right corner set increasing*, written RCSI $(X_1, X_2, \cdots X_n)$ if $P[X_1 > x_1, \cdots X_n > x_n \mid X_1 > x_1', \cdots X_n > x_n']$ is nondecreasing in $x_1', \cdots, x_n'$ for every choice of $x_1, \cdots x_n$. This generalises some notions of dependence that were studied by Lehmann [4] and Esary and Proschan [3]. Setting $\bar{F}(x_1, x_2, \cdots x_n) = P[X_1 > x_1, X_2 > x_2, \cdots, X_n > x_n]$ we have:

DEFINITION. A distribution function $F(x_1, x_2, \cdots, x_n)$ on the nonnegative orthant is *multivariate* IHR if it satisfies the conditions:

(i) $\bar{F}(x_1+t, \cdots, x_n+t)/\bar{F}(x_1, \cdots, x_n) \leqq \bar{F}(x_1'+t, \cdots x_n'+t)/\bar{F}(x_1', \cdots, x_n')$ for all $x_i \geqq x_i' \geqq 0$ and all $t \geqq 0$.

(ii) RCSI $(X_1, \cdots X_n)$.

REMARKS. a. In the context of life-testing, condition (i) is essentially a "wear-out" condition analogous to the univariate case.

b. Condition (ii) characterises a positive dependence between the random variables which has much intuitive appeal if one thinks of the X_i as lifetimes of devices in a common environment with corresponding high or low stress levels.

c. It is of interest to know what possible distribution functions can satisfy (i) when the inequality sign is replaced by an equality. The general form of such a distribution in the bivariate case was determined by Marshall and Olkin [5], and they showed further that if the marginal distributions are exponential, then the distribution is the bivariate exponential distribution: $\bar{F}(x_1, x_2) = \exp[-(\lambda_1 x_1 + \lambda_2 x_2 + \lambda_{12} \max(x_1, x_2))]$.

For the n-dimensional case, the requirement that the $(n-1)$-dimensional marginals be multivariate exponential (MVE) yields the n-dimensional MVE. This result is similar to the univariate case in which the "boundary" of the class of IHR distributions is the class of exponential distributions.

Received September 25, 1969; revised October 10, 1969.

[1] This research was carried out at the University of California, Berkeley, and was partially supported by the Office of Naval Research under Contract Nonr-3656 (18).