

ON A CERTAIN RECURRENCE RELATION

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1. Introduction. Let x_1, x_2, \dots, x_n be a random sample of n observations drawn from the truncated Poisson distribution

$$(1) \quad f(x; \theta) = \alpha \theta^x / x!, \quad x \in T$$

where $\alpha = 1/[\exp(\theta) - 1]$, $0 < \theta < \infty$, and T is the set of positive integers. The minimum variance unbiased estimator $g(t, n)$ for the parameter θ in (1), based on a sample size n and sample total t , has been obtained by Roy and Mitra [2] in terms of the numbers $\Delta^n O^t$ where Δ is the forward difference operator, while Tate and Goen [3] have obtained $g(t, n)$ in the form

$$(2) \quad \begin{aligned} g(t, n) &= tC(t, n)/n && \text{for } t > n \\ &= 0 && \text{for } t = n \end{aligned}$$

where $C(t, n) = 1 - [S(t-1, n-1)/S(t, n)]$, and $S(t, n)$ is the Stirling number of the second kind with arguments t and n as defined by Riordan ([1] page 33). It may be pointed out that the numbers $S(t, n)$ are related to the numbers $\Delta^n O^t$ by means of $n! S(t, n) = \Delta^n O^t$.

The minimum variance unbiased estimator $g(t, n)$ defined by (2) has been tabulated to three decimal places by Roy and Mitra [2] for $n = 2(1)10$ and $t = 2(1)96$, whereas Tate and Goen [3] have furnished a table, correct to five decimal places, of the function $C(t, n)$ for $n = 2(1)49$ and $t = 3(1)50$, using Miksa's unpublished table of Stirling numbers of the second kind. Tate and Goen point out (see [3] page 760) that an extended table of $C(t, n)$ would be quite useful but they have been unable to devise a method for computing $C(t, n)$ which does not depend on entries in the table of Stirling numbers of the second kind.

The purpose of this note is to provide a recurrence relation for $C(t, n)$ which does not involve Stirling numbers of the second kind and which would be quite useful for either extending the existing table of $C(t, n)$ or tabulating the function $g(t, n)$ itself.

2. Recurrence relation for $C(t, n)$. The definition of $C(t, n)$ in (2) gives us

$$(3) \quad C(t+1, n) = 1 - [S(t, n-1)/S(t+1, n)].$$

Using the recurrence formula for Stirling numbers of the second kind available in Riordan ([1] page 33) as

$$(4) \quad S(t+1, n) = S(t, n-1) + nS(t, n)$$

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