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ON STANTON AND MULLIN'S CONSTRUCTION OF ROOM SQUARES[1]

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1. Introduction. A *Room square* is a $(2n+1) \times (2n+1)$ array in which each cell is either empty or contains an (unordered) pair of the $2n+2$ symbols $0, 1, \cdots, 2n, \infty$. Moreover, each of the symbols appears exactly once in each row and in each column of the array, and each (unordered) pair appears exactly once in the entire array. A *cyclic Room square* (CRS) is a Room square in which the entries a_{ij}, $i, j = 0, 1, \cdots, 2n$, satisfy $a_{ij} = (x, y)$ if and only if $a_{i-1, j-1} = (x-1, y-1)$ (where addition is reduced modulo $2n+1$ and $\infty + a = a + \infty = \infty$ for $a = 0, 1, \cdots, 2n$). A *patterned Room square* (PRS) is a CRS in which the first row (the entries a_{0i}, $i = 0, 1, \cdots, 2n$) contains the (unordered) pairs $(\infty, 0)$, $(1, 2n)$, $(2, 2n-1), \cdots$, $(n, n+1)$, not necessarily in this order. Stanton and Mullin [3] gave a computer construction for PRS of (odd) side 7 through 49, with the exception of 9, for which a PRS does not exist. This note extends that result by showing that PRS of side p always exist, where p is a prime not of the form 2^s+1.

Mullin and Nemeth [1] give the following definitions for a starter and adder for a general finite Abelian group G of order $2n+1$: A *starter* in G is a set $X = \{(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)\}$ of unordered pairs of elements of G such that (i) the elements $x_1, y_1, x_2, y_2, \cdots, x_n, y_n$, comprise all nonzero elements of G, and (ii) the differences $\pm(x_i - y_i)$, $i = 1, 2, \cdots, n$ comprise all nonzero elements of G (generating each exactly once). An *adder for* X is a set $A(X)$ of n distinct nonzero elements $a_1, a_2, \cdots, a_n$ from G such that the elements $\{x_i + a_i, y_i + a_i\}$, $i = 1, 2, \cdots, n$ are all distinct and comprise all the nonzero elements of G.

As shown in [3] and in greater detail in [1], the existence of a starter and adder for a cyclic group of order $2n+1$ implies the existence of a CRS of side $2n+1$. The proof of this fact uses the following procedure for constructing a CRS from a starter X and adder $A(X)$: the entries a_{0i} of the first row of a $(2n+1) \times (2n+1)$ array are specified by (i) $a_{00} = (\infty, 0)$, and (ii) $a_{0, -a_i} = (x_i, y_i)$, $i = 1, 2, \cdots, n$, and then the array is completed by imposing the condition that the array must satisfy the cyclic property. The result is a CRS. A PRS is thus determined by an adder which corresponds to a starter consisting of the pairs $(1, 2n)$, $(2, 2n-1), \cdots, (n, n+1)$. In the next section, an adder is given for such a starter.

Other references to the literature of Room squares will be found in [1].

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